Instruction Manual and Experiment Guide

# PROJECTILE LAUNCHER AND BALLISTIC PENDULUM

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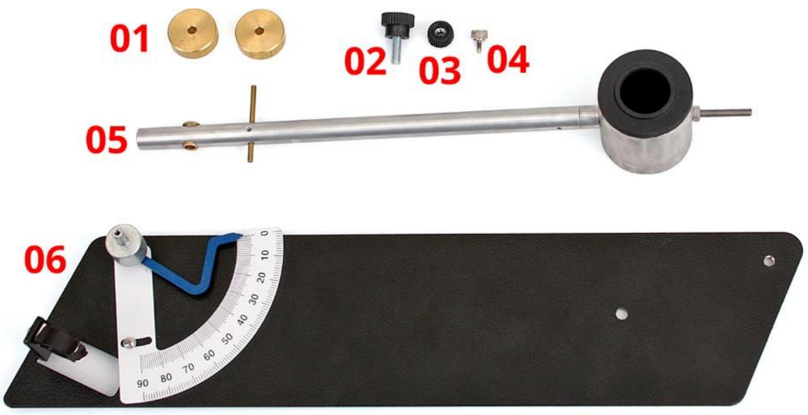
# COMPOSITION


## Projectile Launcher

| Item | Code | Quant. | Unit | Description |
|---|---|---|---|---|
| 01 | 62002176 | 01 | Un. | CLAMP "C" |
| 02 | 62005751 | 01 | Un. | CANNON HOLDER |
| 03 | 62002015 | 01 | Un. | CANNON (PROJECTILE LAUNCHER) |
| 04 | 62001074 | 02 | Un. | STEEL BALL Ø25MM |
| 05 | 62005120 | 01 | Un. | PLASTIC BALL Ø25MM |
| 06 | 62005177 | 01 | Un. | TUBE FOR CANNON COMPRESSION |
| 07 | 48005003 | 02 | Un. | BUTTERFLY NUT (CANNON'S FASTENER) |
| 08 | 50001004 | 02 | Un. | FLAT WASHER (CANNON'S FASTENER) |
| 09 | 62005317 | 01 | Un. | CANNON'S FASTENER |
| 10 | 62001023 | 01 | Un. | SENSORS/SPHERE SHOCK'S HOLDER |
| 11 | 62002056 | 01 | Un. | BALL'S MAGNETIC FIXING (FOR COLLISION) |
| 12 | 53003001 | 01 | Un. | FASTENER FOR BALL'S MAGNETIC FIXING |
| 13 | 03003011 | 01 | Un. | TAPE MEASURE 05M |
| 14 | 62005274 | 01 | Un. | PLUMB BOB WITH MAGNETIC FIXING |




## Ballistic Pendulum Accessories (sold separately)

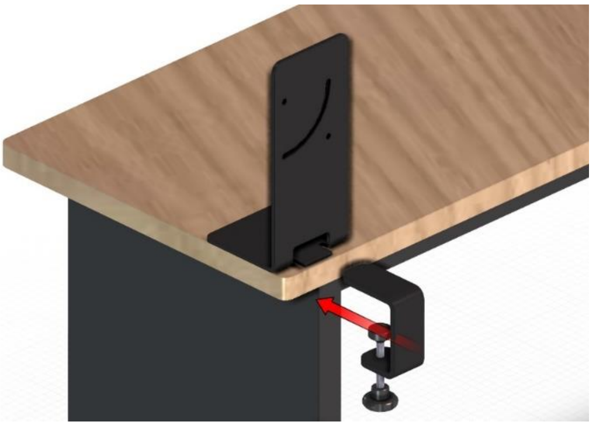
| Item | Code | Quant. | Unit | Description |
|---|---|---|---|---|
| 01 | 62005611 | 02 | Un. | 50G CYLINDER MASS |
| 02 | | 01 | Un | PENDULUM’S TOWER THUMB SCREW |
| 03 | 53003001 | 01 | Un. | PENDULUM’S TOWER THUMB SCREW’S NUT |
| 04 | 53001009 | 01 | Un. | PENDULUM’S THUMB SCREW |
| 05 | 62002055 | 01 | Un. | BALLISTIC PENDULUM |
| 06 | 62002182 | 01 | Un. | PENDULUM’S TOWER |

# ACCESSORIES (SOLD SEPARATELY)

| Code | Quant. | Unit | Description | Picture |
|---|---|---|---|---|
| 62001226 | 01 | Un. | DIGITAL TIMER AZB-30 USB |  |
| 62001255 | 01 | Un. | PHOTOGATE TIMER LITE |  |
| 62001201 | 02 | Un. | PHOTOELECTRIC SENSOR PGS-D10 |  |
| 04002037 | 01 | Un. | FLIGHT TIME SENSOR TFS-D10 |  |
| 30002014 | 06 | Un. | PAIR OF DOCKABLE RODS Ø12,7MMX400MM |  |
| 29003002 | 04 | Un. | BIG TRIPOD |  |
| 31003001 | 04 | Un. | METALLIC CLIP AZB-027 |  |
| 02005006 | 04 | Un. | LAB CLAMP WITH RING |  |

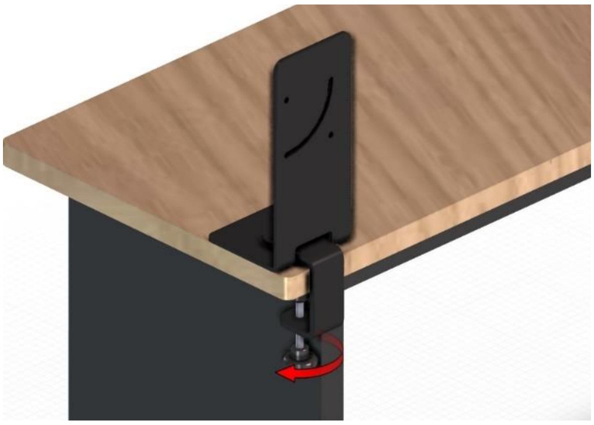
# ASSEMBLY SCHEMES

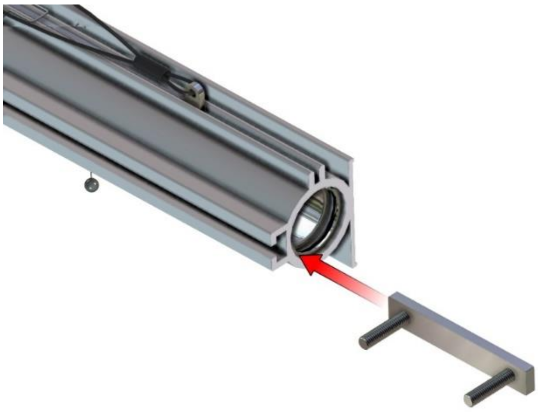
## PROJECTILE LAUNCHER



Position the CANNON HOLDER on the corner of a table. Then insert the clamp "C" as shown in the figure.

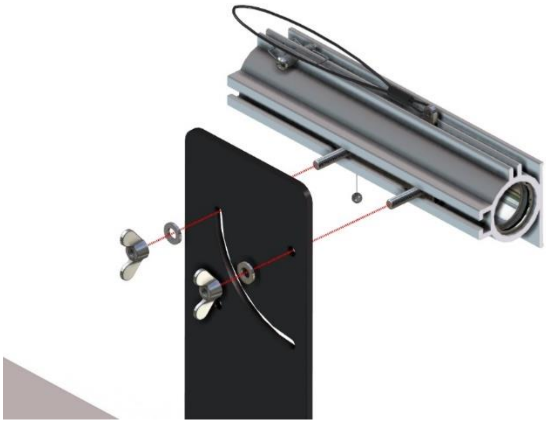
Tighten the clamp knob until the CANNON HOLDER is steady.

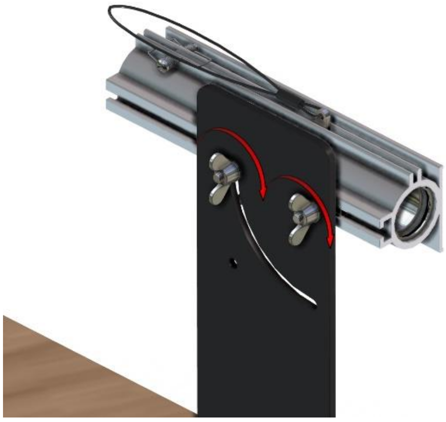




Position the CANNON’S FASTENER inside the cannon rail.

Place the cannon in the support "L" as shown in the figure and fasten it with FLAT WASHER and BUTTERFLY NUT (CANNON’S FASTENER) without tightening to the end.

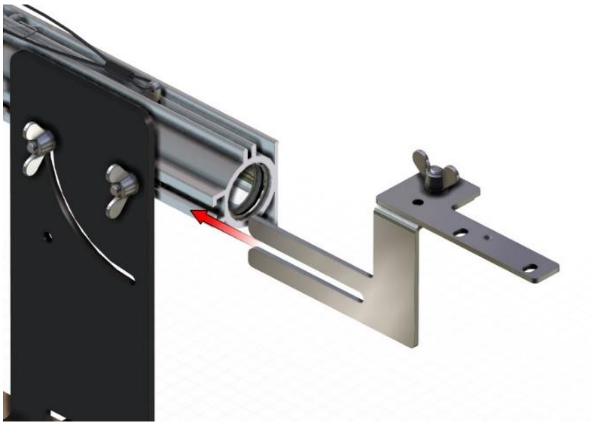




Position the cannon at the desired angle and then tighten the BUTTERFLY NUT.

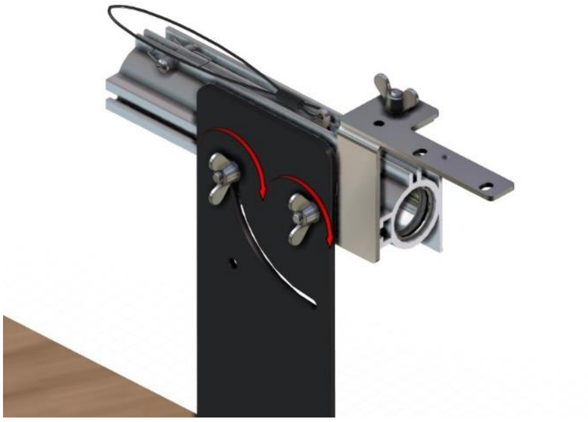
Ready, the cannon is assembled and ready to be used.

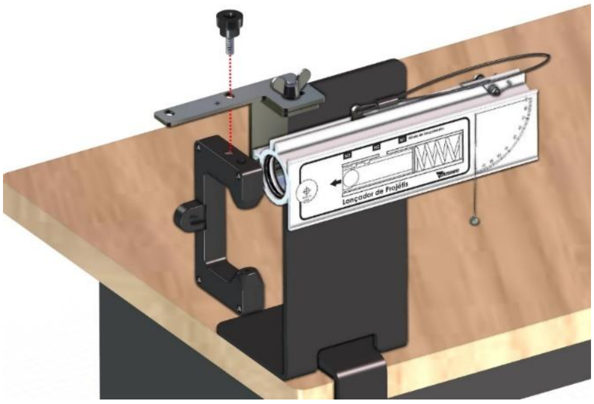
# PHOTOELECTRIC SENSORS PGS-D10



Loosen the BUTTERFLY NUT slightly and insert the SENSORS/SPHERE SHOCK'S HOLDER as shown in figure.

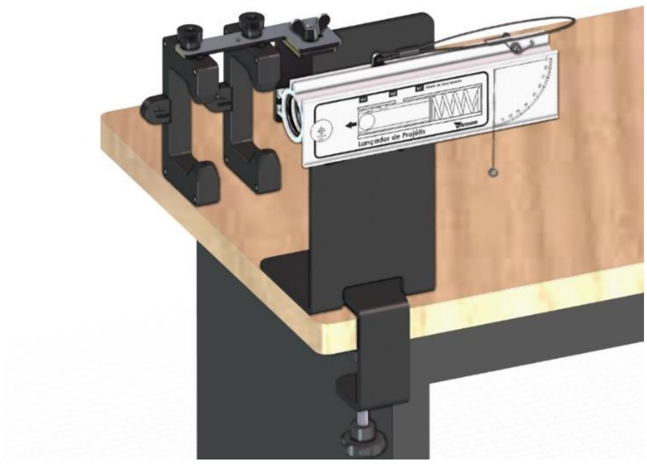
With the support in place, tighten the BUTTERFLY NUT again.

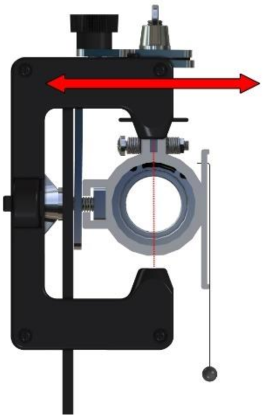




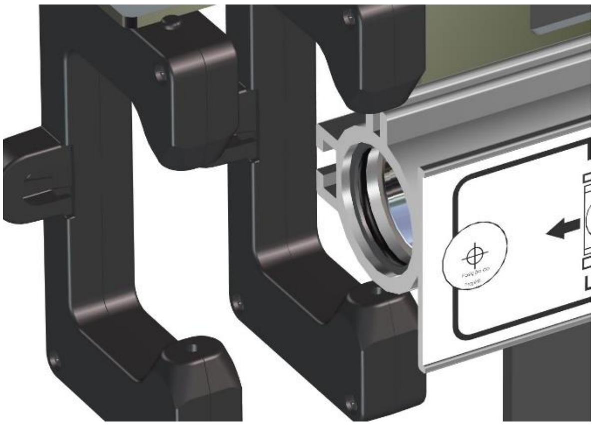
Fix the sensors as shown in the figure.

The image on the right shows the two sensors assembled.



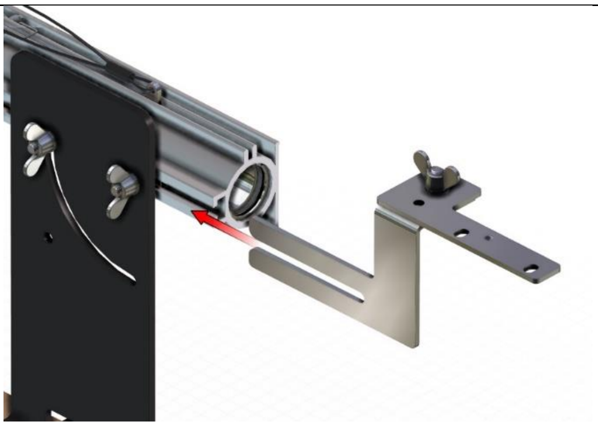


The figure shows the central alignment of the sensor.



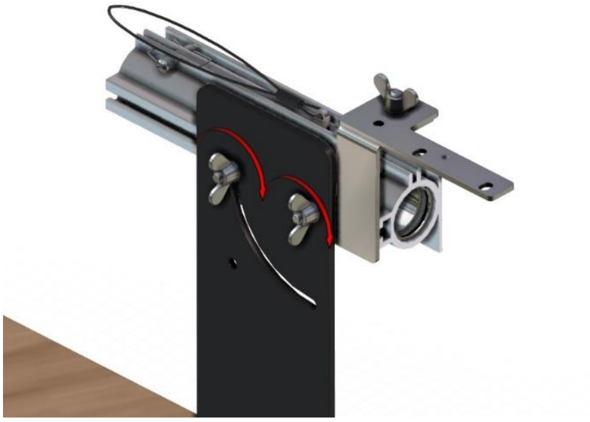
The image shows the positioning of the timer "start" sensor. It should be well positioned at the exit of the cannon

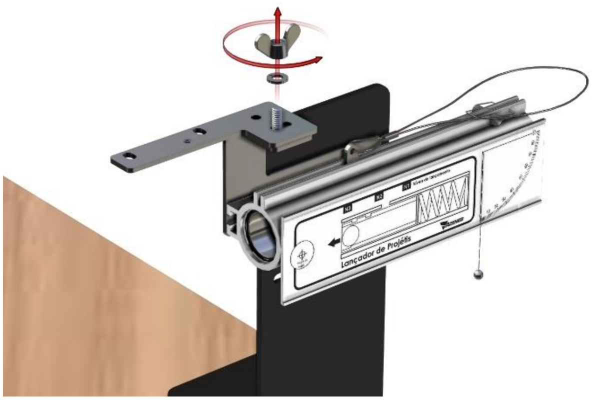
# BALL’S MAGNETIC FIXING



Loosen the BUTTERFLY NUT slightly and insert the SENSORS/SPHERE SHOCK’S HOLDER as shown in figure.

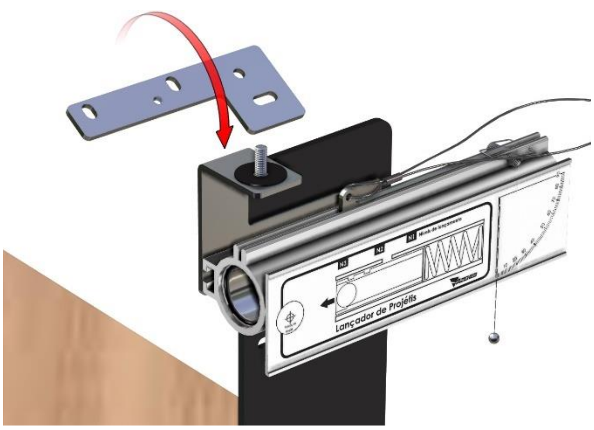
Then tighten the BUTTERFLY NUT again.

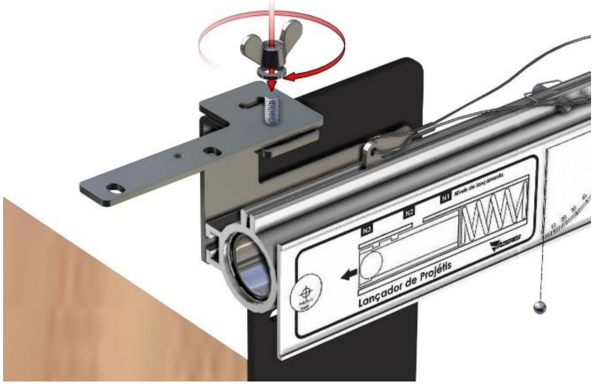




Remove the BUTTERFLY NUT to release the "L" shaped part..

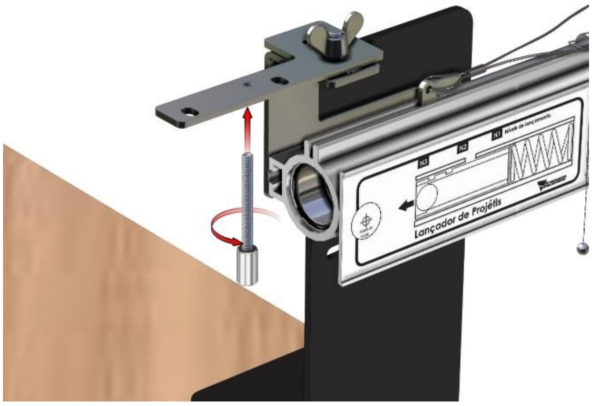
Reverse its position.

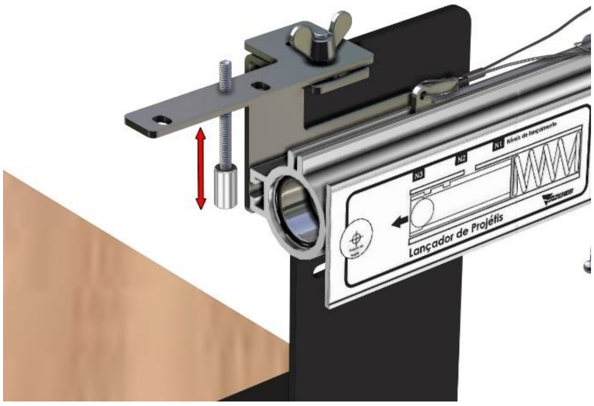




Replace the BUTTERFLY NUT.

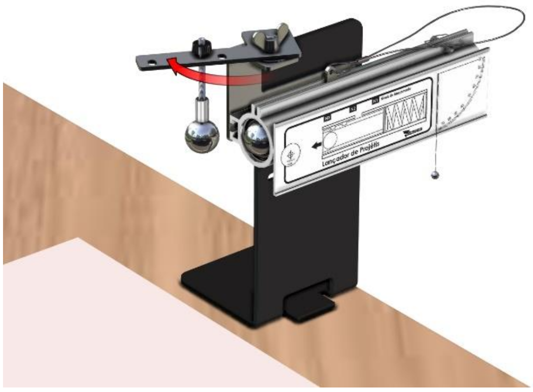
Thread the BALL’S MAGNETIC FIXING.



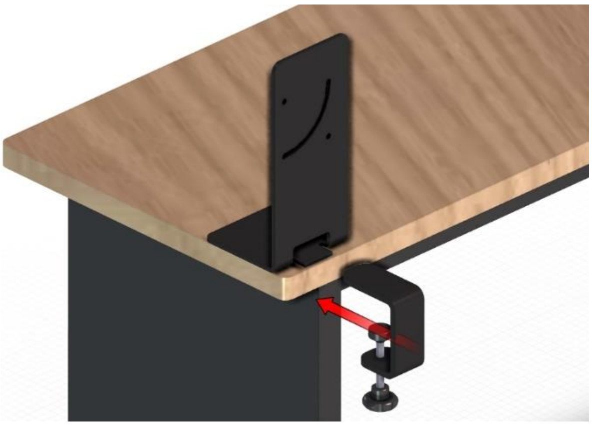


Adjust the height of BALL’S MAGNETIC FIXING. If necessary place one ball in the cannon and another in the fixing to check the alignment. The two balls must be aligned by a center line.

To adjust the collision angle, loosen the butterfly nut slightly and then move the support "L" to the desired position and retighten the butterfly nut.

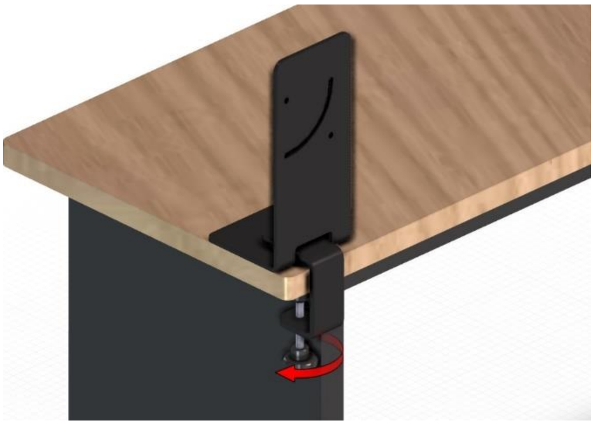


# BALLISTIC PENDULUM



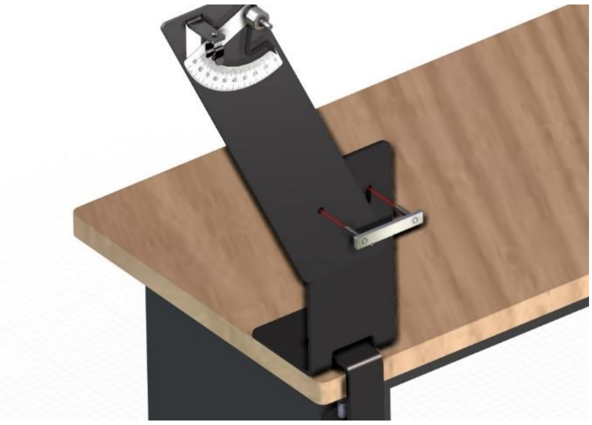
Position the CANNON HOLDER in one of the corners of the table and fit the "C" clamp. If the table surface is slick, use a thin rubber cloth under the HOLDER.

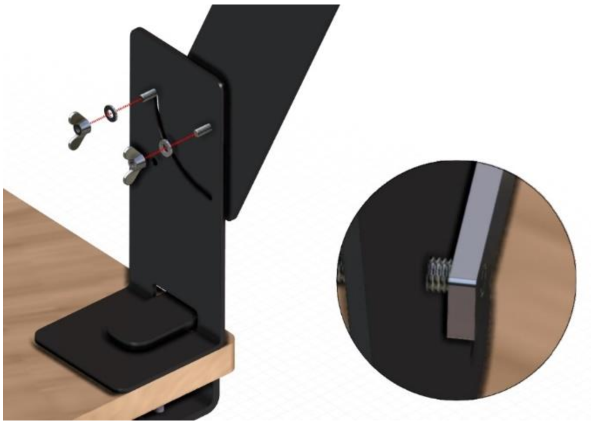
Tighten "C" clamp .



Position the PENDULUM’S TOWER.

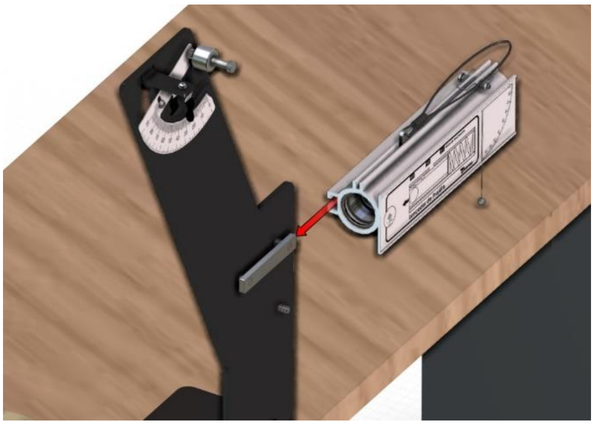
Insert the CANNON’S FASTENER into the holes indicated in the figure.





Fasten the washers and butterfly nuts. Do not tighten until the end, it is necessary to leave a gap to fit the cannon.

Lock the PENDULUM'S TOWER by fastening the PENDULUM'S TOWER THUMB SCREW as shown in the figure.

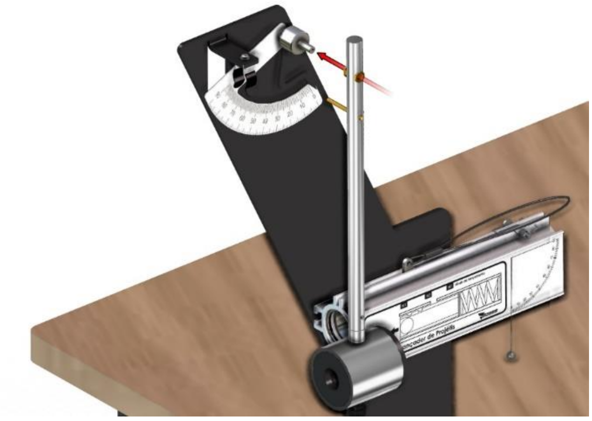


Slide the cannon as shown in the figure.

Tighten the butterfly nuts to fix the cannon.

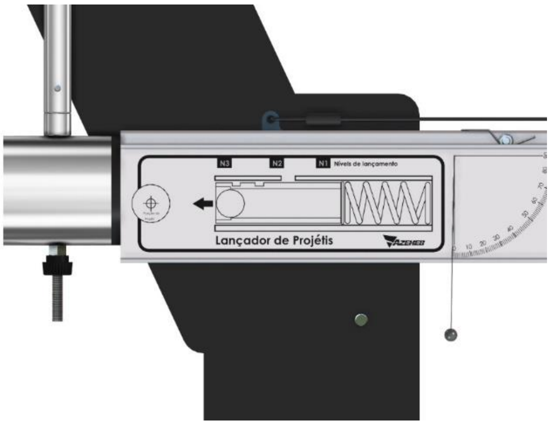
Remove the PENDULUM’S THUMB SCREW.

Place the BALLISTIC PENDULUM.



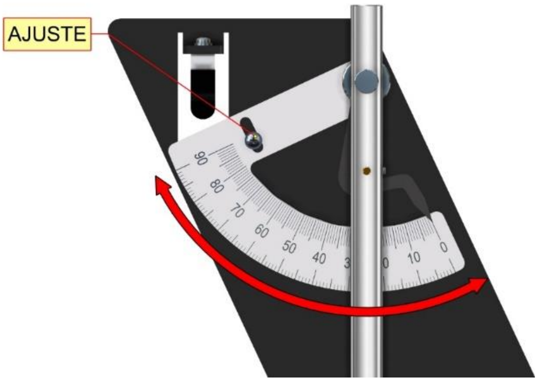
Reattach the PENDULUM’S THUMB SCREW.

Position the cannon launcher.



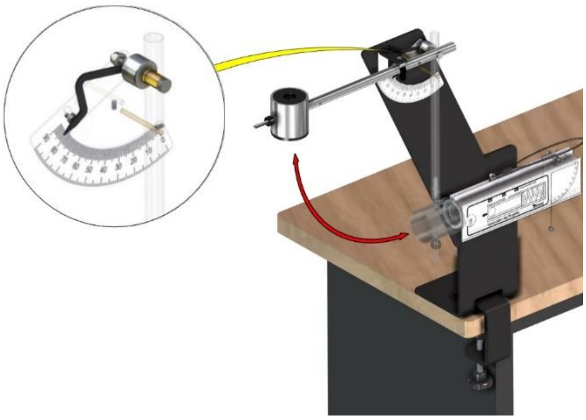

The positioning must be as seen in the figure.

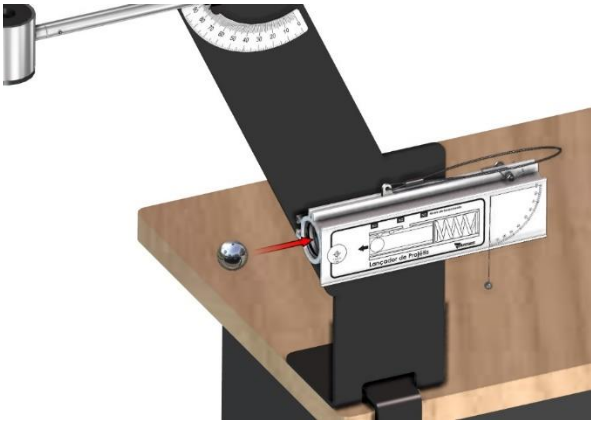
Observe ballistic pendulum alignment.




Turn the angle marker pointer until it touches the ballistic pendulum arm. If angle is not 0º, loosen the screw and adjust the scale to 0º.

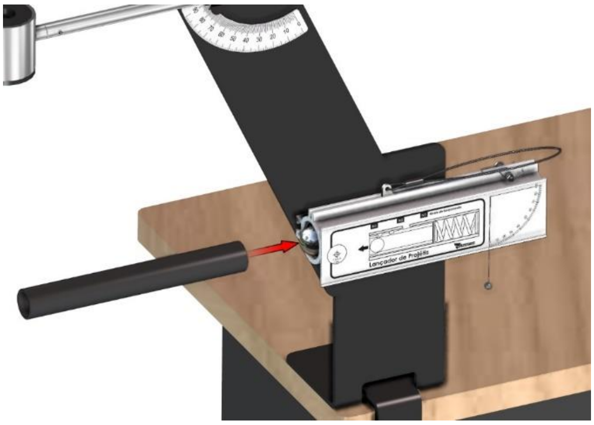
With the ballistic pendulum assembled and adjusted, raise the pendulum until the arm attaches to the cramp at the top.





Place the ball on the cannon.

Using the plastic tube, push the ball to charge the cannon.



With the cannon charged, lower the ballistic pendulum until it snaps into the cannon's mouth and position the angle indicating pointer at 0º.
Done, now just shoot the cannon.

# EXPERIMENTS

## EXPERIMENT 01 – HORIZONTAL PROJECTILE LAUNCH

Objectives:
- Recognizing the physical quantities involved in a horizontal projectile launch
- Checking the relationship between the physical quantities present in a horizontal launch.

>>> ***Material Used*** >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

| *Item* | *Code* | *Quant.* | *Unit* | *Description* |
|---|---|---|---|---|
| *01* | *62002176* | *01* | *Un.* | *CLAMP "C"* |
| *02* | *62005751* | *01* | *Un.* | *CANNON LAUNCHER HOLDER* |
| *03* | *62002015* | *01* | *Un.* | *CANNON (PROJECTILE LAUNCHER)* |
| *05* | *62005120* | *01* | *Un.* | *PLASTIC BALL Ø25MM* |
| *06* | *62005177* | *01* | *Un.* | *TUBE FOR CANNON COMPRESSION* |
| *07* | *48005003* | *02* | *Un.* | *BUTTERFLY NUT (CANNON'S FASTENER)* |
| *08* | *50001004* | *02* | *Un.* | *FLAT WASHER (CANNON'S FASTENER)* |
| *09* | *62005317* | *01* | *Un.* | *CANNON'S FASTENER* |
| *10* | *62001023* | *01* | *Un.* | *SENSORS/SPHERE SHOCK'S HOLDER* |
| *13* | *03003011* | *01* | *Un.* | *TAPE MEASURE 05M* |
| *14* | *62005274* | *01* | *Un.* | *PLUMB BOB WITH MAGNETIC FIXING* |
| *XX* | *62001226* | *01* | *Un.* | *DIGITAL TIMER AZB-30 USB (*)* |
| *XX* | *62001201* | *02* | *Un.* | *PHOTOELECTRIC SENSOR PGS-D10 (*)* |
| *XX* | *04002037* | *01* | *Un.* | *FLIGHT TIME SENSOR TFS-D10* |

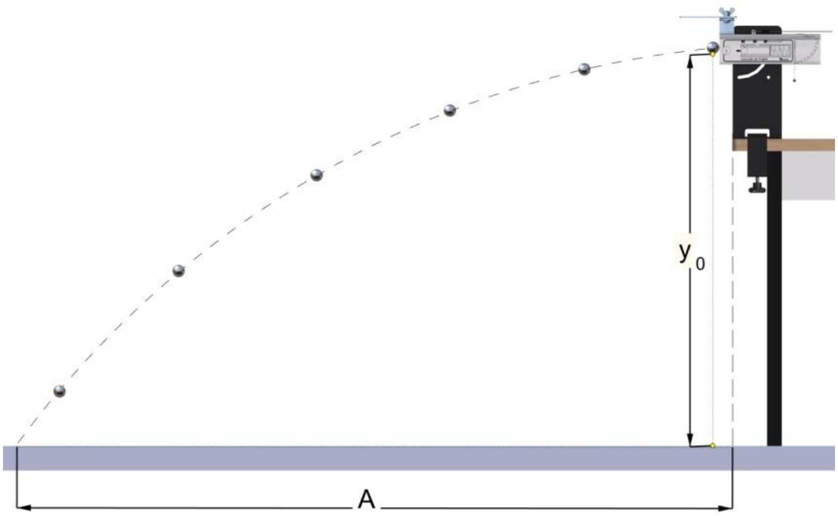
**(*) It does not accompany the product. It is sold separately.**

### Part I - Determining the horizontal launch velocity ($v_0$) using the reach measure (A) and the launch height ($y_0$)

>>> ***Experimental Procedures***>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

Do not look directly at the cannon exit, as it may be charged!

**1.** Assemble the launcher so that the projectile has space to move and fall on the floor, as shown in the figure.



**2.** Measure the launch height ($y_o$) in relation to the floor. The measurement shall be made from the lower part of the ball indicated at the exit of the cannon, to the floor, according to the figure.

$y_o$= 0,923 m

**3.** With a plumb bob, mark on a paper sheet (scotch taped on the floor) the position (origin of horizontal displacement). The plumb bob must match the vertical passing through the center of the ball.

**4.** Place the plastic ball in the cannon and compress the spring to the second stage. Pull the trigger and observe where the ball touches the floor.

**5.** At the dropping point of the ball place a paper sheet (scotch tape it as well) with a carbon paper over it.

**6.** Repeat the launch 10 times and measure the reached distance range A using the measure tape. Complete the table 1 with the values.

Table 1

| N | Reached distance (m) |
|---|---|
| 1 | 1,780 |
| 2 | 1,782 |
| 3 | 1,781 |
| 4 | 1,785 |
| 5 | 1,820 |
| Average value | 1,790 |

**>>> *Analysis of Results and Conclusions* >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>**

**7.** What are the active forces in the sphere after the launch?

In the projectile only the gravitational force acts, since the friction with the air is considered insignificant.

**8.** Classify the projectile motion according to the two directions (vertical and horizontal).

As the gravitational force is considered constant and has the direction of the vertical of the place, the vertical motion will be a Non-uniform linear motion, with local gravity acceleration, g. As friction with air is considered insignificant, horizontally there is no force acting on the projectile, and the motion is a uniform linear motion.

**9.** Which equation must be used in each motion?

- In horizontal motion: $\begin{cases} x = x_o + v_{0x} \cdot t \\ v_x = v_{ox} = constant \end{cases}$

- In vertical motion: $\begin{cases} y = y_o + v_{oy} \cdot t + \frac{a_y \cdot t^2}{2} \\ v_y = v_{oy} + a_y \cdot t \end{cases}$

**10.** Use the horizontal launch equations and calculate the time in the air.

When touching the floor, we have to the vertical motion: $\begin{cases} y = 0 \\ v_{oy} = 0 \\ a = -g \end{cases} \rightarrow t_{ar} = \sqrt{\frac{2y_o}{g}}$

$$t_{ar} = \sqrt{\frac{2 \times 0,923}{9,78}} = 0,434\ s$$

Substituting $t_{ar}$ into the horizontal motion equation (with $x_o = 0$), we obtain the expression that provides the launch velocity:

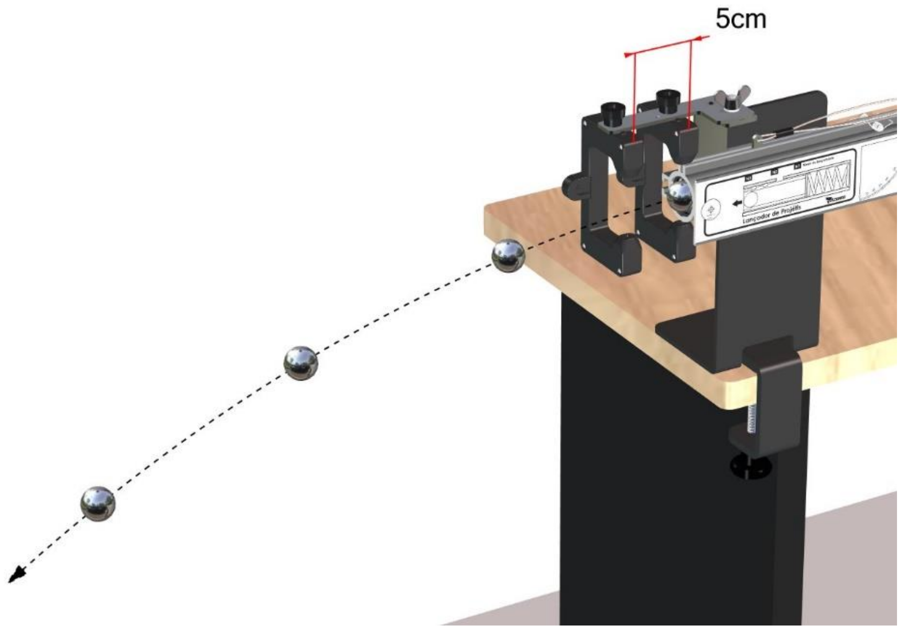
$$v_{o1} = v_{ox} = \frac{A}{t_{ar}} = \frac{1,79}{0,434} \rightarrow v_{o1} = 4,12\ m/s$$

# Part II - Determination of horizontal velocity of launch ($v_o$) using the measurement of projectile passage time by two sensors

***>>> Experimental Procedures>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>***

Do not look directly at the cannon exit, as it may be charged!

**1.** Assemble the projectile launcher as shown in the figure (sensors are positioned approximately 5cm apart):




**2.** Align correctly the sensors.

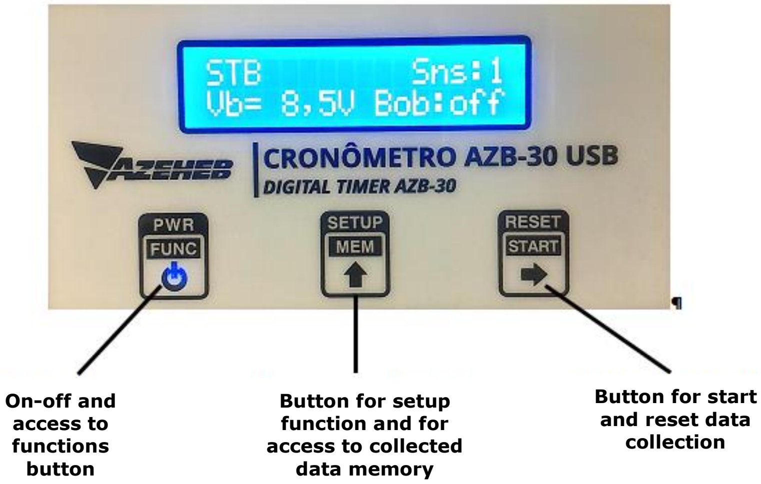
**3.** Connect the sensors to the timer.

**4.** Measure the distance between the two sensors centers.

$\Delta x=0{,}050$ m

**5.** Turn on the timer and check if it has identified the sensors. Test the sensors by interrupting the infrared passage. An intermittent exclamation mark (!) should appear in the upper right corner of the screen indicating that the sensors are identified.

– When you turn on the timer, it displays the STANDBY screen and should show:
- STB
- Vb: 8,5V – Output Voltage.
- Sns :2– Number of sensors identified.
- Bobbin: off - indicating the coil is off.




6. Select function 1 by clicking FUNC. The display will show the function 1 F1. The display should read "[F1]" "Btn nS:2". In this function the timer will be using two sensors, and the chronometer will present two time measurements. (See information in the timer manual).


7. Schedule the timer for data acquisition by starting the time counting on the first sensor and interrupting the second one.

8. Press SETUP, press for a few seconds until the display shows CFG, use the key (↓) and adjust from Btn to Sns. Click START to end programming. On the nS: 1 screen, it indicates that the timer will only take one measure of time.


9. As the objective is only to measure the time interval between the two sensors, collect the ball just after passing through the second sensor, using a cardboard box.

10. Position the plastic ball in the second stage of the launcher.

**11.** By pressing START on the display it will show the "standby" signal (*) flashing. By pressing START again, the signal (*) is in the operating mode, waiting for the ball to pass through the sensor.

**12.** After the launch the screen shows the time interval value of the ball passing through the two sensors.

**13.** Write down the time measured in the table 1. Repeat data collection at least 5 times.

**14.** By pressing START the time measurement of the screen is sent to memory. For a new measure go back to item 11. Complete the table 1.

Table 1

| N | Δx (m) | Δt (s) | *v (m/s)* |
|---|---|---|---|
| 1 | 0,05 | 0,01230 | 4,0650 |
| 2 | 0,05 | 0,01229 | 4,0683 |
| 3 | 0,05 | 0,01219 | 4,1017 |
| 4 | 0,05 | 0,01209 | 4,1356 |
| 5 | 0,05 | 0,01218 | 4,1051 |
| | Average value | | 4,0952 |

>>> ***Analysis of Results and Conclusions*** >>>>>>>>>>>>>>>>>>>>>>>>>>>>

**15.** Use the equation below and calculate the ball velocity through the sensors:

$$v = \frac{\Delta x}{\Delta t}$$

**16.** Determine the average value of the launch velocity.

$$v_{o2} = \frac{4,0650 + \cdots + 4,1051}{5} = \frac{20,4757}{5} \rightarrow v_{02} = 4,0952\ m/s$$

**17.** Compare the experimental values of the launch velocities found by processes (a) and (b). Was there any significant difference between the values found?

| *Method* | *By the reach measure and launch height ($v_{o1}$)* | *By the time of passage between two sensors ($v_{o2}$)* |
|---|---|---|
| *Velocity (m/s)* | *4,12* | *4,095* |

**18.** Calculate the average launch velocity of the two procedures.

$$v_{launch} = v_o = \frac{4,12 + 4,095}{2} = 4,108\ m/s$$

**19.** Calculate the percentage error: $\boldsymbol{e}\% = \frac{|\boldsymbol{v_{01}} - \boldsymbol{v_{02}}|}{\frac{\boldsymbol{v_{01}} + \boldsymbol{v_{02}}}{2}}$

$$\boldsymbol{e}\% = \frac{|\mathbf{4,12 - 4,095}|}{\frac{\mathbf{4,12 + 4,095}}{\mathbf{2}}} = \mathbf{0,61}\%$$

It is observed that the difference between the two values found is lower than the error tolerance of 5% and can be considered practically equal.

**20.** Justify possible inconsistencies between results.

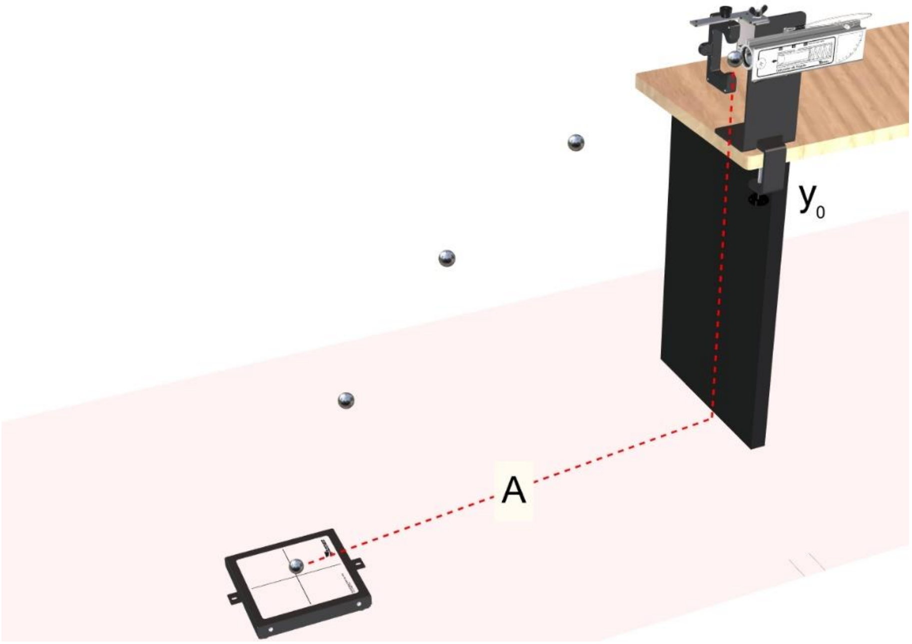
Measurement of time intervals, distances and friction of the ball with air.

# Part III - Determination of horizontal velocity of launch ($v_o$) using the measure of the reach (A) and the time of the projectile in the air

***>>> Experimental Procedures>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>***

Do not look directly at the cannon exit, as it may be charged!

**1.** Assemble the cannon as shown in the figure (with a sensor and the flight time sensor). Cover the flight sensor with a carbon paper to mark the point where the ball touches the lowest point of the path.




**2.** Connect the sensor and platform to the timer.

**3.** Adjust the sensor position as close as possible to the launcher's mouth.

**4.** Turn on the timer and check if it has identified the sensors and the platform. Test the sensors by interrupting the infrared passage. An intermittent exclamation mark (!) Should appear in the upper right corner of the screen indicating that the sensor 1 are identified. Tap the sensor briefly, the display shows signal 2 (!) indicating that the timer has identified the flight sensor.

– When you turn on the timer, it displays the STANDBY screen and should show:
– STB
– Vb: 8,5V – Output Voltage.
– Sns :2– Number of sensors identified.
– Bobbin: off - indicating the coil is off.

**5.** Select function 1 by clicking FUNC. The display will show the function 1 F1. The display should read "[F1]" "Btn nS:2". In this function the timer will be using two sensors, and the timer will present on time measurement. (See information in the timer manual).


**6.** Schedule the timer for data acquisition by starting the time counting on the first sensor and interrupting the second one.

**7.** Press SETUP, press for a few seconds until the display shows CFG, use the key (↓) and adjust from Btn to Sns. Click START to end programming. On the nS: 1 screen, it indicates that the timer will only take one measure of time.


**8.** Position the plastic ball in the second stage of the cannon.

**9.** By pressing START on the display it will show the "standby" signal (*) flashing. By pressing START again, the signal (*) is in the operating mode, waiting for the ball to pass through the sensor.

**10.** After the ball touches the flight sensor, the time counting stops.

**11.** Write down in the table the measured time. Repeat the procedure at least 5 times and calculate the average reach distance (A). To measure the reach, observe the following precautions:

- Place a plumb bob from the center of the sensor to the floor. Mark this reference.
- The reach shall be measured from the reference point on the floor to the point where the ball touches the platform.

**12.** By pressing START the time measurement of the screen is sent to memory. For a new measure go back to item 11. Complete the table 1.

Table 1

| N | A (m) | Δt (s) | v (m/s) |
|---|---|---|---|
| 1 | 1,71 | 0,42321 | 4,041 |
| 2 | 1,65 | 0,40094 | 4,115 |
| 3 | 1,77 | 0,42649 | 4,150 |
| 4 | 1,67 | 0,41267 | 4,047 |
| 5 | 1,66 | 0,40915 | 4,057 |
| | Average value | | 4,082 |

### >>> *Analysis of Results and Conclusions* >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

**13.** Use the follow equation and calculate the ball velocity through the sensors:

$$v = \frac{A}{\Delta t}$$

**14.** Determine the average value of the launch velocity.

$$v_{o3} = \frac{4,041 + \cdots + 4,057}{5} = \frac{20,410}{5} \rightarrow v_{03} = 4,082\ m/s$$

**15.** Compare the experimental values of the launch velocities found by processes (a) and (b). Was there any significant difference between the values found?

| *Method* | *By the reach measure and launch height ($v_{o1}$)* | *By the time of passage between two sensors ($v_{o2}$)* | *By the residence time in the air ($v_{o3}$)* |
|---|---|---|---|
| *Velocity (m/s)* | *4,12* | *4,095* | *4,082* |

**16.** Calculate the average value of the launch velocity found by the two procedures.

$$v_{launch} = v_o = \frac{4,120 + 4,095 + 4,082}{2} = 4,099\ m/s$$

# EXPERIMENT 02 – PROJECTILE OBLIQUE LAUNCH

**Objectives:**
**- Recognizing the physical quantities involved in a projectile oblique launch.**
**- Check the relationship among the physical quantities present in an oblique launch.**

>>> ***Material Used*** >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

| *Item* | *Code* | *Quant.* | *Unit* | *Description* |
|---|---|---|---|---|
| *01* | *62002176* | *01* | *Un.* | *CLAMP "C"* |
| *02* | *62005751* | *01* | *Un.* | *CANNON LAUNCHER HOLDER* |
| *03* | *62002015* | *01* | *Un.* | *CANNON (PROJECTILE LAUNCHER)* |
| *05* | *62005120* | *01* | *Un.* | *PLASTIC BALL Ø25MM* |
| *06* | *62005177* | *01* | *Un.* | *TUBE FOR CANNON COMPRESSION* |
| *07* | *48005003* | *02* | *Un.* | *BUTTERFLY NUT (CANNON'S FASTENER)* |
| *08* | *50001004* | *02* | *Un.* | *FLAT WASHER (CANNON'S FASTENER)* |
| *09* | *62005317* | *01* | *Un.* | *CANNON'S FASTENER* |
| *10* | *62001023* | *01* | *Un.* | *SENSORS/SPHERE SHOCK'S HOLDER* |
| *13* | *03003011* | *01* | *Un.* | *TAPE MEASURE 05M* |
| *14* | *62005274* | *01* | *Un.* | *PLUMB BOB WITH MAGNETIC FIXING* |
| *XX* | *62001226* | *01* | *Un.* | *DIGITAL TIMER AZB-30 USB (*)* |
| *XX* | *62001201* | *02* | *Un.* | *PHOTOELECTRIC SENSOR PGS-D10 (*)* |
| *XX* | *04002037* | *01* | *Un.* | *FLIGHT TIME SENSOR TFS-D10* |

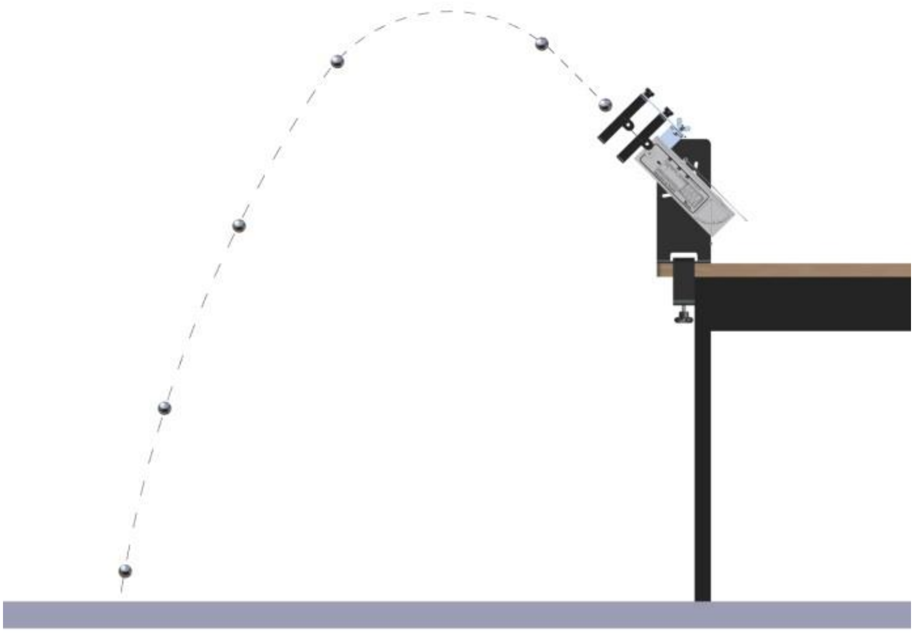
**(*) It does not accompany the product. It is sold separately.**

## Part I - Measure of Reach (A) and determining of launch velocity ($v_0$) using the measurement of the passage time between two sensors.

>>> ***Experimental Procedures***>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

Do not look directly at the cannon exit, as it may be charged!

**1.** Assemble the equipment as shown (the space between sensors is 5 cm):



**2.** Align sensors correctly and position the first of them as close as possible to the launcher's exit.

**3.** Connect the sensors to the timer.

**4.** Measure the distance between the two sensors centers.

$\Delta x$=0,050 m

**5.** Adjust the cannon launch position to an angle of 30º.

**6.** Measure the launch height ($y_0$) (from the bottom of the ball to the floor).

$y_0$= 0,975 m

**7.** Launch the ball once to put a paper sheet and a carbon paper over it.

**8.** Select function 1 by clicking FUNC. The display will show the function 1 F1. The display should read "[F1]" "Btn nS:2". In this function the timer will be using two sensors, and the timer will present two time measurements. (See information in the timer manual).


**9.** Schedule the timer for data acquisition by starting the time counting on the first sensor and interrupting the second one.

**10.** Press SETUP, press for a few seconds until the display shows CFG, use the key (↓) and adjust from Btn to Sns. Click START to end programming. On the nS: 1 screen, it indicates that the timer will only take one measure of time.


**11.** Position the plastic ball in the second stage of the launcher.

**12.** By pressing START on the display it will show the "standby" signal (*) flashing. By pressing START again, the signal (*) is in the operating mode, waiting for the ball to pass through the sensor.

**13.** Fire the launcher. After the launch the screen shows the time interval value of the ball passing through the two sensors.

**14.** Measure the horizontal range of the ball (A) and the time interval value and note this data in the table. Repeat data collection at least 5 times.

**15.** By pressing START the time measurement of the screen is sent to memory. For a new measure go back to item 12. Complete the table 1.

Table 1

| N | A(m) | $t_p$ (s) | $v_o$ (m/s) |
|---|---|---|---|
| 1 | 2,53 | 0,01219 | 4,1017 |
| 2 | 2,58 | 0,01195 | 4,1841 |
| 3 | 2,57 | 0,01226 | 4,0783 |
| 4 | 2,49 | 0,01244 | 4,0193 |
| 5 | 2,47 | 0,01246 | 4,0128 |
| Average value | 2,53 | 0,01228 | 4,0792 |

### >>> *Analysis of Results and Conclusions* >>>>>>>>>>>>>>>>>>>>>>>>>>>>

**16.** Use the values in the table and calculate the average value of the measured reach - $A_{measured}$

$$A_{med} = \frac{\sum A_i}{N}$$

$$A_{med} = \frac{2,53 + \cdots + 2,47}{5} = 2,53\ m$$

**17.** Use the equation below and table values to calculate the ball velocity through the sensors:

$$v = \frac{\Delta x}{\Delta t}$$

**18.** Determine the average value of the launch velocity ($v_o$).

$$v_o = \frac{4,1017 + \cdots 4,0128}{5} = \frac{20,3962}{5} \rightarrow v_0 = 4,0792\ m/s$$

**19.** Use the values obtained for the initial velocity ($v_o$), height ($y_o$) and launch angle ($\theta$) and determine the predicted reach value using the equation:

$$A = v_0 \cdot cos\theta \cdot \frac{v_0 \cdot sen\theta + \sqrt{v_0^2 \cdot sen^2\theta + 2gy_0}}{g}$$

$$A_{predicted} = 4,0792 \cdot cos30^o \cdot \frac{4,0792 \cdot sen30^o + \sqrt{4,0792^2 \cdot sen^2 30^o + 2 \times 9,78 \times 0,975}}{9,78}$$

$$A_{predited} = 2,48\ m$$

**20.** Compare the predicted reach value with the measured value.

$$e\% = \frac{\left|A_{measured} - A_{predicted}\right|}{\frac{A_{measured} + A_{predicted}}{2}}$$

$$e\% = \frac{|2,53 - 2,48|}{\frac{2,53 + 2,48}{2}} \times 100 = 2,0\%$$

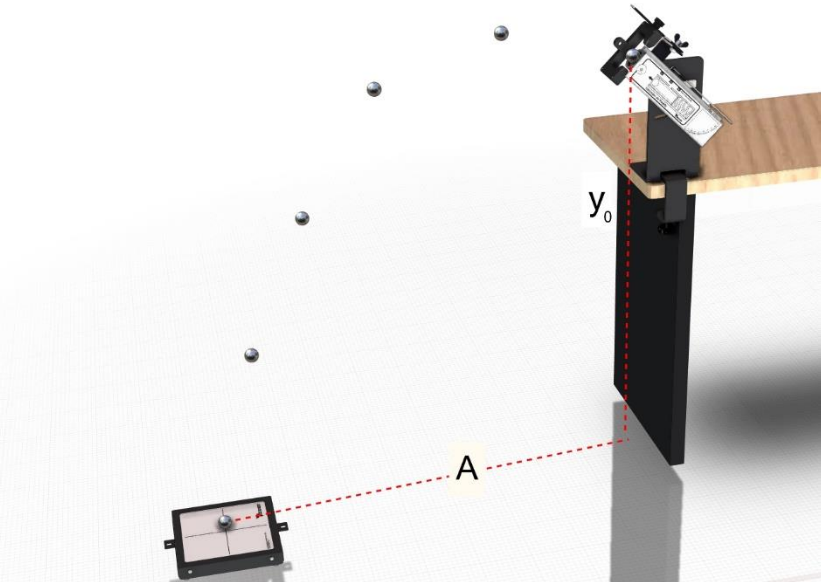
Values can be considered equal.

## Part II – Determining launch velocity ($v_o$) using the measure of reach (A) and the projectile residence time interval in the air.

>>> ***Experimental Procedures***>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

Do not look directly at the cannon exit, as it may be charged!

**1.** Assembly as shown in the figure (with a sensor and the flight time sensor).




**2.** Connect the sensors and the platform to the timer.

**3.** Adjust the sensor position as close as possible to the launcher's mouth.

**4.** Move the assembly to a launch at an angle of 60º.

**5.** Turn on the timer and check if it has identified the sensors and the platform. Test the sensors by interrupting the infrared passage. An intermittent exclamation mark (!) should appear in the upper right corner of the screen indicating that the sensor 1 are identified. Tap the sensor briefly; the display shows signal 2 (!) indicating that the timer has identified the flight sensor.

– When you turn on the timer, it displays the STANDBY screen and should show:
– STB
– Vb: 8,5V – Output Voltage.
– Sns :2– Number of sensors identified.
– Bobbin: off - indicating the coil is off.

**6.** Select function 1 by clicking FUNC. The display will show the function 1 F1. The display should read "[F1]" "Btn nS:2". In this function the timer will be using two sensors, and the chronometer will present two time measurements. (See information in the timer manual).


**7.** Schedule the timer for data acquisition by starting the time counting on the first sensor and interrupting the second one.

**8.** Press SETUP, press for a few seconds until the display shows CFG, use the key (↓) and adjust from Btn to Sns. Click START to end programming. On the nS: 1 screen, it indicates that the timer will only take one measure of time.


**9.** Position the plastic ball in the second stage of the launcher and perform some launches to identify the probable position of the platform.

**10.** By pressing START on the display it will show the "standby" signal (*) flashing. By pressing START again, the signal (*) is in the operating mode, waiting for the ball to pass through the sensor.

**11.** After the ball touches the flight sensor, the time counting stops.

**12.** Note the time interval value provided by the timer in the table.

**13.** Measure the reach (A) and note it in the table. To measure the reach, observe the following precautions:

- Place a plumb bob from the center of the sensor to the floor. Mark this reference.
- The reach shall be measured from the reference point on the floor to the point where the ball touches the platform.

**14.** Repeat data collection at least 5 times.

**15.** By pressing START the time measurement of the screen is sent to memory. For a new measure go back to item 10.

| N | A (m) | Δt (s) | $v_{ox}$ (m/s) | $v_o$(m/s) |
|---|---|---|---|---|
| 1 | 1,91 | 0,93204 | 2,049 | 4,099 |
| 2 | 1,92 | 0,93626 | 2,051 | 4,101 |
| 3 | 1,93 | 0,93796 | 2,058 | 4,115 |
| 4 | 1,93 | 0,93085 | 2,073 | 4,147 |
| 5 | 1,94 | 0,93934 | 2,065 | 4,131 |
| | Average value | | | 4,119 |

>>> ***Analysis of Results and Conclusions*** >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

**16.** Use the follow equation and calculate the ball horizontal velocity ($v_{ox}$):

$$v_{ox} = \frac{A}{\Delta t}$$

$v_{ox} = \frac{1{,}91}{0{,}93204} = 2{,}049\ m/s$ para a primeira linha da tabela

**17.** Determine the average value of the launch velocity ($v_o$) for each measure carried out.

$$v_o = \frac{v_{ox}}{cos60^o}$$

$v_o = \frac{2{,}049}{cos60^o} = 4{,}099\ m/s$, para a primeira linha da tabela

**18.** Determine the average value of the launch velocity ($v_{om}$).

$$v_{o3} = \frac{4{,}099 + \cdots + 4{,}131}{5} = 4{,}119\ m/s$$

**19.** Compare the experimental values of the launch velocities found by processes (a) and (b). Was there any significant difference between the values found?

| *Method* | *By the time of passage between two sensors ($v_{o2}$)* | *By the residence time in the air ($v_{o3}$)* |
|---|---|---|
| *Velocity (m/s)* | *4,079* | *4,119* |

**20.** Compare the results.

$$e\% = \frac{|4{,}079 - 4{,}119|}{\frac{4{,}079 + 4{,}119}{2}} \times 100\% = 0{,}98\%$$

# EXPERIMENT 03 – RELATIONSHIP BETWEEN REACH AND LAUNCH ANGLE

**Objective:**
**- Checking the dependence between a projectile's reach and launch angle.**

>>> ***Material Used*** >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

| *Item* | *Code* | *Quant.* | *Unit* | *Description* |
|---|---|---|---|---|
| *01* | *62002176* | *01* | *Un.* | *CLAMP "C"* |
| *02* | *62005751* | *01* | *Un.* | *CANNON LAUNCHER HOLDER* |
| *03* | *62002015* | *01* | *Un.* | *CANNON (PROJECTILE LAUNCHER)* |
| *05* | *62005120* | *01* | *Un.* | *PLASTIC BALL Ø25MM* |
| *06* | *62005177* | *01* | *Un.* | *TUBE FOR CANNON COMPRESSION* |
| *07* | *48005003* | *02* | *Un.* | *BUTTERFLY NUT (CANNON'S FASTENER)* |
| *08* | *50001004* | *02* | *Un.* | *FLAT WASHER (CANNON'S FASTENER)* |
| *09* | *62005317* | *01* | *Un.* | *CANNON'S FASTENER* |
| *10* | *62001023* | *01* | *Un.* | *SENSORS/SPHERE SHOCK'S HOLDER* |
| *13* | *03003011* | *01* | *Un.* | *TAPE MEASURE 05M* |
| *14* | *62005274* | *01* | *Un.* | *PLUMB BOB WITH MAGNETIC FIXING* |

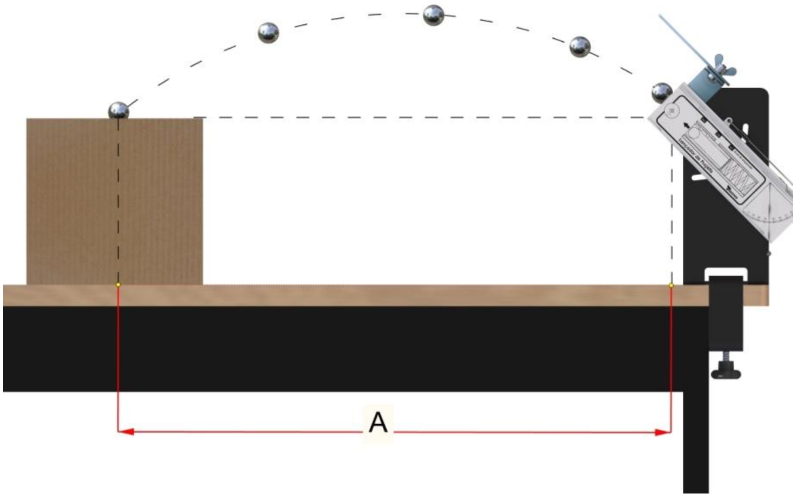
**(*) It does not accompany the product. It is sold separately.**

## Part I – Launch without gap

>>> ***Experimental Procedures***>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

Do not look directly at the cannon exit, as it may be charged!

**1.** Assemble the launcher as shown.




**2.** Provide a platform (for example, a box) that allows the projectile to reach it at the same horizontal point as the launch point.

**3.** Adjust the launch angle to 10º and insert the plastic ball into the second stage.

**4.** Make some previous launches to locate the point where the ball will touch the surface of the platform.

**5.** Fix a paper sheet on the platform and a carbon paper over

**6.** Make three launches and measure the value of launch position A (m) to the point where the ball touches the platform.

**7.** Repeat the launch procedures for the angles suggested in table 1.

Table 1

| Angle | Reach 1 | Reach 2 | Reach 3 | Reach measured | Reach calculated |
|---|---|---|---|---|---|
| 10 | 0,600 | 0,590 | 0,594 | 0,595 | 0,594 |
| 20 | 1,120 | 1,110 | 1,109 | 1,113 | 1,116 |
| 30 | 1,498 | 1,530 | 1,490 | 1,506 | 1,503 |
| 40 | 1,700 | 1,695 | 1,700 | 1,698 | 1,709 |
| 45 | 1,700 | 1,800 | 1,750 | 1,750 | 1,736 |
| 50 | 1,690 | 1,680 | 1,730 | 1,700 | 1,709 |
| 60 | 1,490 | 1,510 | 1,480 | 1,493 | 1,503 |
| 70 | 1,100 | 1,180 | 1,200 | 1,160 | 1,116 |
| 80 | 0,598 | 0,59 | 0,614 | 0,601 | 0,594 |

**>>> *Analysis of Results and Conclusions* >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>**

**8.** Use the experimental values from table 1 and calculate the average reach distance for each launch angle.

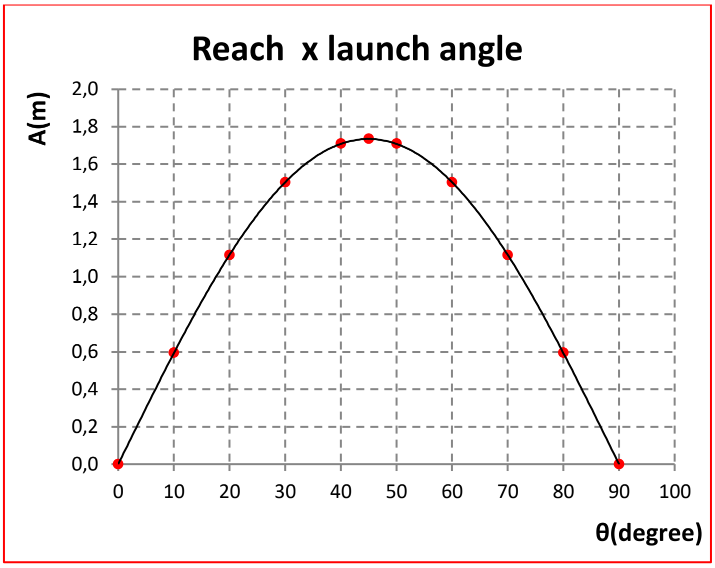
**9.** Use launch velocity equal to 4.12 in expression:

$$A_{calc} = \frac{v_0^2 \times sen2\theta}{g}$$

And obtain the calculated reach value for each angle.

$A_{calc1} = \frac{4{,}12^2 \times sen(2\times 10^0)}{9{,}78} = 0{,}594$, to the first row of the table.

**10.** Draw the reaches graph ($A_{measured}$) according to the launch angle.




**11.** According to the table and the graph, what is the launch angle value that provides the longest reach?

The angle that provides the longest reach is 45º.

**12.** What can be concluded about the reach value for the complementary angles, that is, 10º and 80º, 20º and 70º, 30º and 60º, 40º and 50º?

Within the error tolerance, it can be stated that the reaches are practically the same when launched under complementary angles.

**13.** Are the experimental results consistent with the theory?

As vo and g are constants, the reach equation, $A_{calc} = \frac{v_0^2 \times sen2\theta}{g}$, only depends on the angle value. Considering two complementary angles θ and (90º -θ), we have:

$$\begin{aligned} sen2(90^0 - \theta) &= sen(180^0 - 2\theta) = sen180^0 \cdot \cos(2\theta) - sen(2\theta) \cdot \cos 180^0 \\ &= 0 \cdot cos2\theta - [sen(2\theta) \cdot (-1) = sen2\theta] \end{aligned}$$

And therefore, the projectile reach should be the same when launched under complementary angles.

Taking into consider the error tolerance of 5%, the experiment confirms the theory, according to the results shown in table 2.

For example, for the complementary angles, 30º and 60º:

$$sen60^o = sen120^o = 0,866 \rightarrow A_{30^0} = A_{60^0} = \frac{0,866 v_0^2}{g}$$

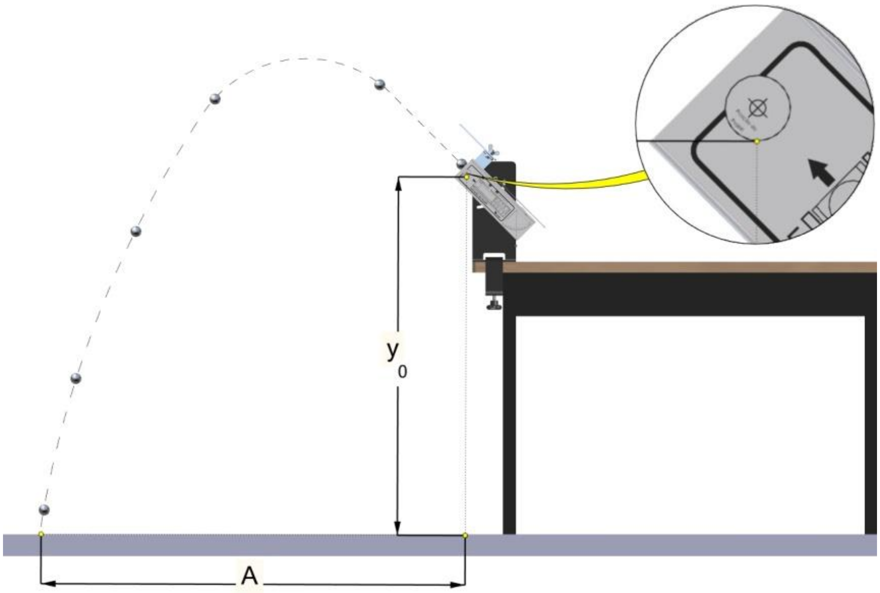
In an analogous way, the same results are obtained for the other pairs of complementary angles.

## Part II – Launch with gap

>>> ***Experimental Procedures***>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

Do not look directly at the cannon exit, as it may be charged!

**1.** Assemble the launcher as shown so that the projectile reaches the floor.




**2.** Use a plumb bob to determine the initial launch position on the floor.

**3.** Adjust the launch angle to 10º and insert the plastic ball into the second stage.

**4.** Fire at least three launches and measure the value of launch position A (m) to the point where the ball touches the floor. Fix a sheet of white paper on the platform and onto it a sheet of carbon paper.

**5.** Fire at least three launches and measure the value of launch position A (m) to the point where the ball touches the floor.

**6.** Repeat the launch procedures for the angles suggested in table 2.

Table 2

| Angle | Reach 1 (m) | Reach 2 (m) | Reach 3 (m) | $A_{average}$ |
|---|---|---|---|---|
| 10° | 2,090 | 2,080 | 2,087 | 2,086 |
| 20° | 2,325 | 2,320 | 2,340 | 2,328 |
| 30° | 2,464 | 2,474 | 2,480 | 2,473 |
| 40° | 2,485 | 2,490 | 2,480 | 2,485 |
| 50° | 2,299 | 2,275 | 2,289 | 2,288 |
| 60° | 1,930 | 1,940 | 1,922 | 1,931 |
| 70° | 1,395 | 1,390 | 1,388 | 1,391 |
| 80° | 0,735 | 0,730 | 0,729 | 0,731 |

**>>> *Analysis of Results and Conclusions* >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>**

**7.** Use the experimental values from table 2 and calculate the average reach value ($A_{medido}$) for each launch angle.

$A_{measured} = \frac{2,090+2,080+2087}{3} = 2,086$ to the first row of the table.

**8.** Use the equation:

$$A_{calculado} = v_0 \cdot cos\theta \cdot \frac{v_0 \cdot sen\theta + \sqrt{v_0^2 \cdot sen^2\theta + 2gy_0}}{g}$$

to obtain the reach calculated value

Table 3

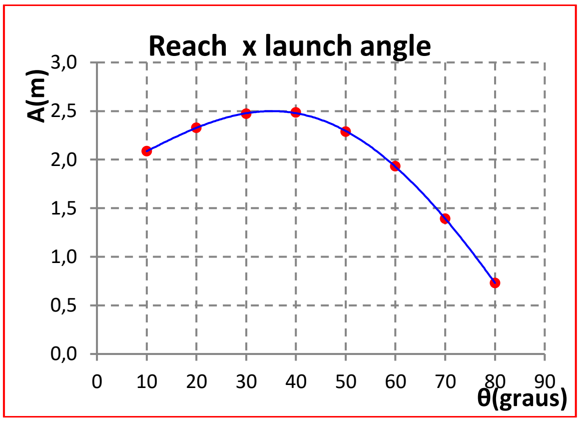
| Angle | $A_{calculated}$ (m) | $A_{measured}$ (m) |
|---|---|---|
| 10º | 2,084 | 2,086 |
| 20º | 2,330 | 2,328 |
| 30º | 2,474 | 2,473 |
| 40º | 2,471 | 2,485 |
| 50º | 2,289 | 2,288 |
| 60º | 1,922 | 1,931 |
| 70º | 1,388 | 1,391 |
| 80º | 0,729 | 0,731 |

$A_{calc} = 4,12 \times cos10^0 \times \frac{4,12 \times sen10^0 + \sqrt{4,12^2 \times sen^2 10^0 + 2 \times 9,78 \times 0,98}}{9,78} = 2,084m$ for the angle of 10º.

**9.** For each of the launch angles, are the reach values (measured and calculated) compatible?

Yes, the difference between their values is small and is within an acceptable tolerance.

**10.** Draw the reaches graph ($A_{measured}$) according to the launch angle.



**11.** According to the table and the graph, what is the launch angle value that provides the longest reach?

The angle that provides the longest reach is about 35º, according to the graph.

**12.** Are the measured and calculated values in agreement with the maximum reach value?

Yes, therefore, although they present a small difference in the reach value, both reach the maximum value for the cited angle, as observed in the graph.

# EXPERIMENT 04 – ANALYSIS OF A TRAJECTORY OF A PROJECTILE

**Objective:**
**- Obtaining the relationship between the vertical and horizontal position of the projectile during its motion.**

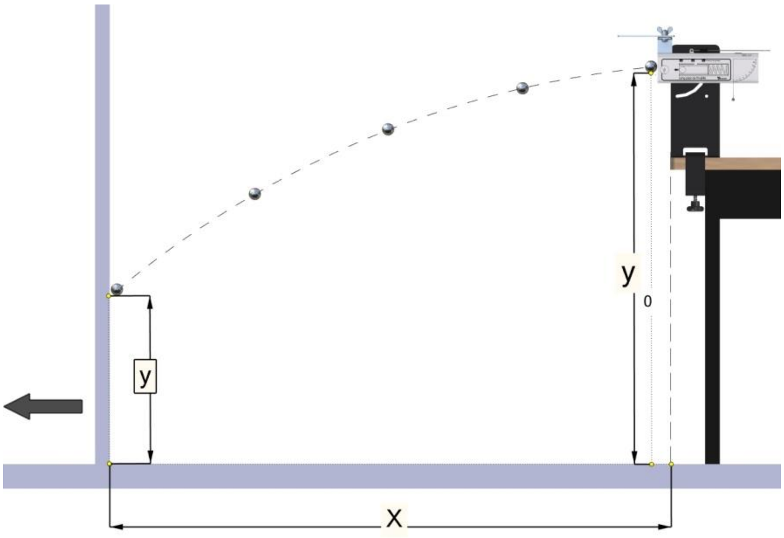
***>>> Material Used >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>***

| Item | Code | Quant. | Unit | Description |
|---|---|---|---|---|
| 01 | 62002176 | 01 | Un. | CLAMP "C" |
| 02 | 62005751 | 01 | Un. | CANNON LAUNCHER HOLDER |
| 03 | 62002015 | 01 | Un. | CANNON (PROJECTILE LAUNCHER) |
| 05 | 62001074 | 01 | Un. | STEEL BALL Ø25MM |
| 06 | 62005177 | 01 | Un. | TUBE FOR CANNON COMPRESSION |
| 07 | 48005003 | 02 | Un. | BUTTERFLY NUT (CANNON'S FASTENER) |
| 08 | 50001004 | 02 | Un. | FLAT WASHER (CANNON'S FASTENER) |
| 09 | 62005317 | 01 | Un. | CANNON'S FASTENER |
| 10 | 62001023 | 01 | Un. | SENSORS/SPHERE SHOCK'S HOLDER |
| 13 | 03003011 | 01 | Un. | TAPE MEASURE 05M |
| 14 | 62005274 | 01 | Un. | PLUMB BOB WITH MAGNETIC FIXING |

## Part I – Horizontal Launch

Do not look directly at the cannon exit, as it may be charged!

***>>> Experimental Procedures>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>***




1. Assemble the launcher as shown.

2. Adjust the cannon for horizontal launches. Use a plumb bob to determine on the floor the horizontal position of the projectile ($x_0 = 0$).

3. Assemble near a wall or set up a vertical bulkhead so that the ball, once launched, collides with it and determines the vertical and horizontal positions of the projectile in the collision with this target, as shown in the figure.

**4.** Position the bulkhead at x = 0.200 m from the horizontal launch position marked on the floor.

**5.** Fix a paper sheet on the target place on the wall and with a carbon paper over it.

**6.** Fire the ball and measure the value of the vertical position in which the projectile hits the target.

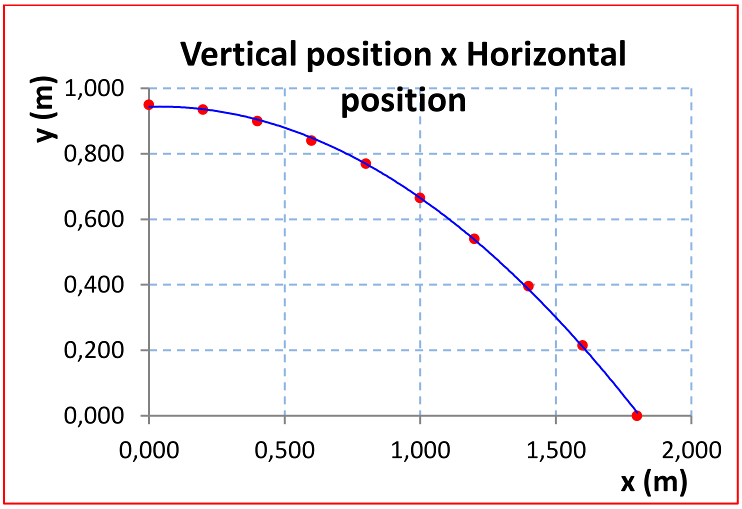
**7.** Reposition the target to the horizontal positions suggested in the table and repeat the launch procedures until you complete it. Complete the table 1.

Table 1

| X (m) | Y (m) | $X^2$ ($m^2$) |
|---|---|---|
| 0,000 | 0,950 | 0,000 |
| 0,200 | 0,935 | 0,040 |
| 0,400 | 0,900 | 0,160 |
| 0,600 | 0,840 | 0,360 |
| 0,800 | 0,770 | 0,640 |
| 1,000 | 0,665 | 1,000 |
| 1,200 | 0,540 | 1,440 |
| 1,400 | 0,395 | 1,960 |
| 1,600 | 0,215 | 2,560 |
| 1,800 | 0,000 | 3,240 |

>>> ***Analysis of Results and Conclusions*** >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

**8.** Use the values collected in table 2 and draw the graph "vertical position (y)" versus "horizontal position (x)".

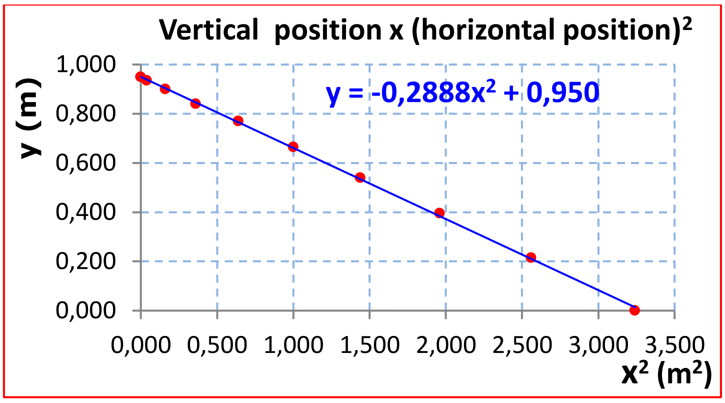



**9.** What is the aspect of the curve obtained?

It is similar to a second-degree parable.

**10.** Do the appropriate change of variables and linearize the graph.

To linearize the graph, we used the vertical position (y) according to the square of the horizontal position (x).




**11.** Obtain the equation that relates the quantities y x

$$y = -0,289x^2 + 0,950 \; (1)$$

**12.** Combine the equations of the two movements performed by the ball and obtain analytically the equation that relates the two positions (y and x) of the ball in its trajectory.

The equations of the motion performed by the ball are:

$$\begin{cases} y = y_o - \frac{g}{2}t^2 \; (v_{0y} = 0) \\ x = v_o \cdot t \end{cases}$$

By combining the two equations, we obtain the equation that relates y and x,y= f(x):

$y = y_0 - \frac{g}{2v_0^2}x^2 \quad (2)$

Comparing (1) and (2), it is verified that they have the same equation.

## Part II – Oblique launch

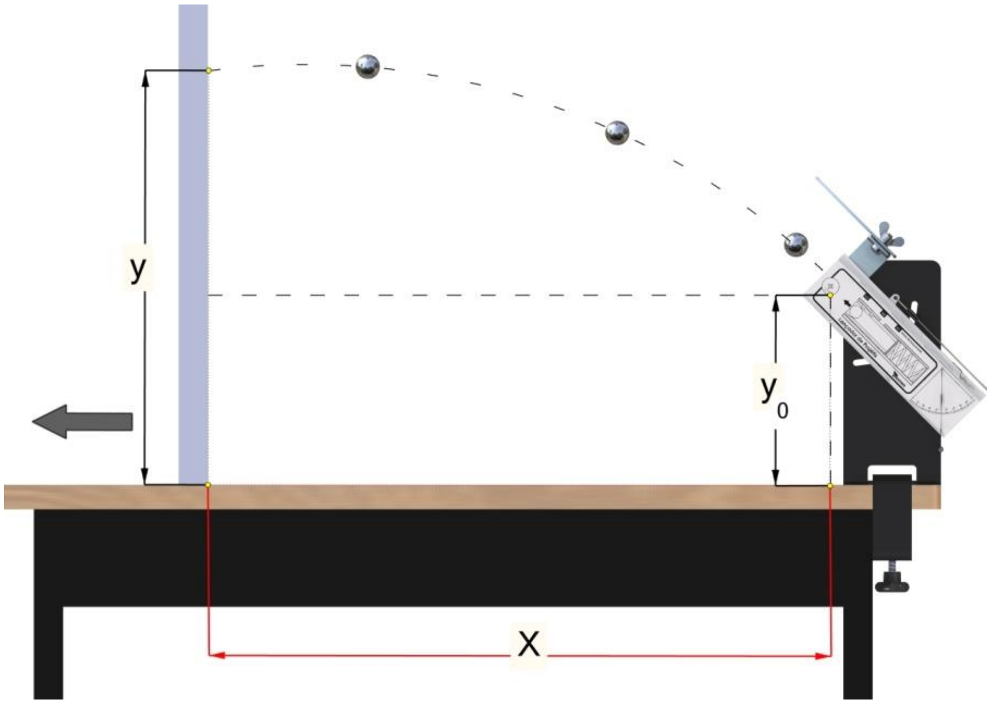
>>> ***Experimental Procedures***>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

Do not look directly at the cannon exit, as it may be charged!

**1.** Assemble the launcher as shown.

**2.** Adjust the launch angle by 60º and measure the height (yo) of the launch point in relation to the tabletop.

**3.** Assemble on the table a vertical bulkhead so that the ball, once launched, collides with it and determines the vertical and horizontal positions of the projectile in the collision with this target, as shown in the figure.




**4.** Use a plumb bob to obtain, on the tabletop, the initial horizontal position ($x_o$=0).

**5.** Use the steel ball and insert it into the second stage of the cannon.

**6.** Position the bulkhead at 0,050 m from the horizontal launch position ($x_o$ = 0) marked on the table top.

**7.** Affix a paper sheet on the target place with a carbon paper over it.

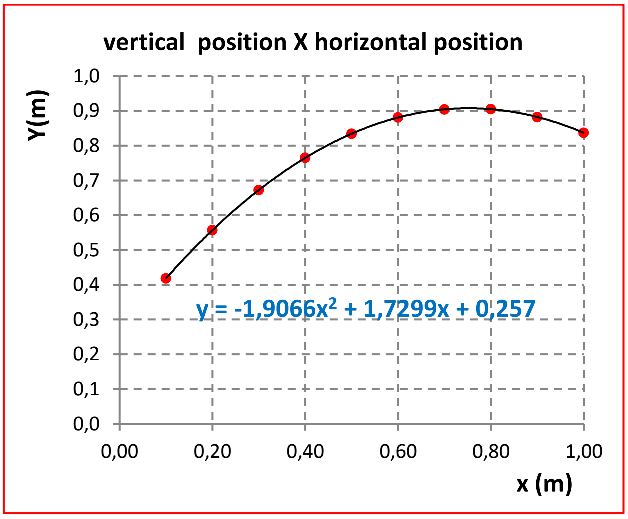
**8.** Reposition the target to the horizontal positions suggested in the table and repeat the launch procedures until you complete it. Complete the table 1.

Table 1

| N | x (m) | y (m) |
|---|---|---|
| 1 | 0 | 0,257 |
| 2 | 0,10 | 0,411 |
| 3 | 0,00 | 0,257 |
| 4 | 0,20 | 0,527 |
| 5 | 0,30 | 0,604 |
| 6 | 0,40 | 0,644 |
| 7 | 0,50 | 0,645 |
| 8 | 0,60 | 0,609 |
| 9 | 0,70 | 0,534 |
| 10 | 0,80 | 0,421 |
| 11 | 0,90 | 0,270 |
| 12 | 1,00 | 0,080 |

>>> ***Analysis of Results and Conclusions*** >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

**9.** Draw the graph "vertical position y" versus "horizontal position x".




**10.** Use the software feature (Excel) and obtain the equation corresponding to the curve.

$$y = -1{,}9066x^2 + 1{,}7299x + 0{,}257$$

**11.** Combine the equations of the motions (vertical and horizontal) and obtain the equation y = f(x).

Horizontal motion: $\boldsymbol{x = v_o \cdot (cos\theta) \cdot t}(1)$

Vertical motion: $\boldsymbol{y = y_o + v_o \cdot (sen\theta) \cdot t - \frac{g}{2} t^2}$ (2)

By changing $\boldsymbol{t = \frac{x}{v_o \cdot cos\theta}}$ from the equation (1) in (2), we have

$$\boldsymbol{y = -\frac{g}{2 \cdot v_o^2 \cdot cos^2\theta} \cdot x^2 + (tg\theta) \cdot x + y_o}$$

**12.** Is the dependence between y and x theoretically found consistent with the equation graphically found?

Yes, the two equations correspond to a complete function of the second degree.

# EXPERIMENT 05 – ANALYSIS OF A TRAJECTORY OF A PROJECTILE II

**Objective:**
**- Verifying the validity of the equation that gives the projectile position at a given moment in an oblique launch.**

>>> ***Material Used*** >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

| *Item* | *Code* | *Quant.* | *Unit* | *Description* |
|---|---|---|---|---|
| *01* | *62002176* | *01* | *Un.* | *CLAMP "C"* |
| *02* | *62005751* | *01* | *Un.* | *CANNON LAUNCHER HOLDER* |
| *03* | *62002015* | *01* | *Un.* | *CANNON (PROJECTILE LAUNCHER)* |
| *05* | *62005120* | *01* | *Un.* | *PLASTIC BALL Ø25MM* |
| *06* | *62005177* | *01* | *Un.* | *TUBE FOR CANNON COMPRESSION* |
| *07* | *48005003* | *02* | *Un.* | *BUTTERFLY NUT (CANNON'S FASTENER)* |
| *08* | *50001004* | *02* | *Un.* | *FLAT WASHER (CANNON'S FASTENER)* |
| *09* | *62005317* | *01* | *Un.* | *CANNON'S FASTENER* |
| *10* | *62001023* | *01* | *Un.* | *SENSORS/SPHERE SHOCK'S HOLDER* |
| *13* | *03003011* | *01* | *Un.* | *TAPE MEASURE 05M* |
| *14* | *62005274* | *01* | *Un.* | *PLUMB BOB WITH MAGNETIC FIXING* |
| *XX* | *62001226* | *01* | *Un.* | *DIGITAL TIMER AZB-30 USB (*)* |
| *XX* | *62001201* | *01* | *Un.* | *PHOTOELECTRIC SENSOR PGS-D10 (*)* |
| *XX* | *04002037* | *01* | *Un.* | *FLIGHT TIME SENSOR TFS-D10* |
| *XX* | *29003002* | *01* | *Un.* | *BIG TRIPOD (*)* |
| *XX* | *30002014* | *01* | *Un.* | *PAIR OF ADAPTABLE STEMS 12,7MM X 400MM (*)* |
| *XX* | *31003001* | *01* | *Un.* | *METALLIC CLIP AZB-027* |

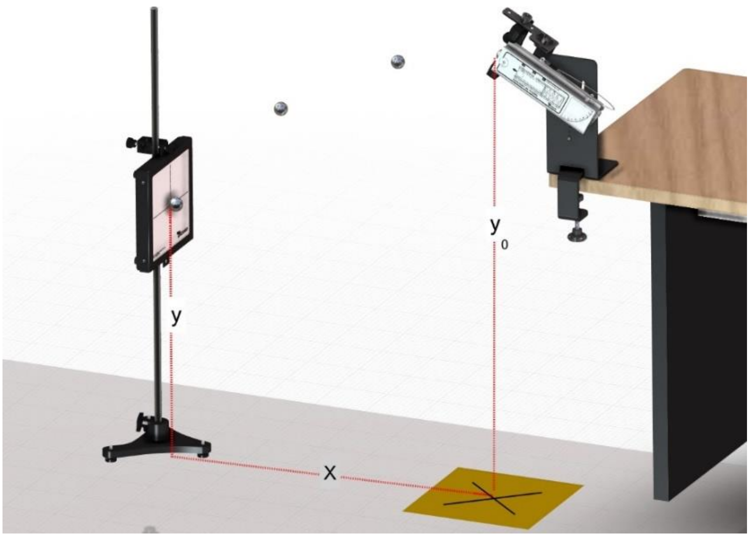
**(*) It does not accompany the product. It is sold separately.**

>>> ***Experimental Procedures***>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

Do not look directly at the cannon exit, as it may be charged!

**1.** Assemble the launcher at an angle of 40 ° to the horizontal on the side of the tabletop, so the projectile has space to move and fall on the floor, as shown in the figure.

**2.** Assemble the flight time sensor as shown.




**3.** Using the plumb bob, affix a paper sheet on the floor aligning with the cannon mouth.

**4.** Measure the launch height (yo) in relation to the floor. The measurement shall be made from the center of the ball indicated at the exit of the cannon, to the floor.

$y_o$= 0,867 m

**5.** Position the flight time sensor at x = 0.400m from the launcher's mouth. Cover the face of the sensor with a carbon paper.

**6.** Place the plastic ball in the projectile launcher and compress the spring to the first stage. Pull the trigger and measure the height y (vertical position) in which the ball touches the platform. Repeat this procedure three times and note in the table the average value of y.

**7.** Reposition the flight time sensor to the positions suggested in the table and repeat the experimental procedures to obtain the values of y.

| N | Horizontal position x (m) | Vertical position y (m) |
|---|---|---|
| 1 | 0,000 | 0,960 |
| 2 | 0,200 | 1,098 |
| 3 | 0,400 | 1,175 |
| 4 | 0,600 | 1,162 |
| 5 | 0,800 | 1,122 |
| 6 | 1,000 | 1,060 |
| 7 | 1,200 | 0,920 |
| 8 | 1,400 | 0,680 |
| 9 | 1,600 | 0,330 |
| 10 | 1,790 | 0,000 |

>>> ***Analysis of Results and Conclusions*** >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

**8.** Combine the oblique launch equations of a projectile and obtain the expression that gives the vertical position y according to the horizontal position x, that is: y = f(x).

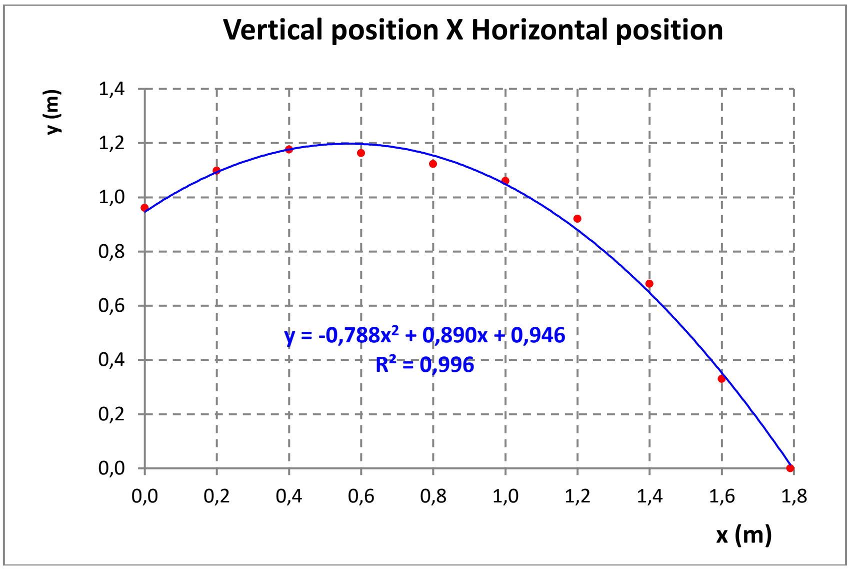
The two-dimensional motion equations in an oblique launch are:

$$\begin{cases} x = v_o \cdot cos\theta \cdot t & (1) \\ y = y_o + v_o \cdot sen\theta \cdot t - \frac{g}{2}t^2 & (2) \end{cases}$$

By isolating $t$ in (1): $t = \frac{x}{v_o cos\theta}$ and substituting in 2, we obtain:

$$y = y_o + x \cdot tg\theta - \frac{g}{2v_o^2 cos^2\theta}x^2$$

**9.** With the experimental data of y and x draw the graph y versus x.




**10.** What does the graph y = f(x) look like?

The curve appears to be a parabola..

**11.** Obtain the equation that represents the curve obtained in the graph.

The application (Excel) displays the equation:
$y = -0,788x^2 - 0,890x + 0,946$

**12.** Does the equation obtained experimentally agree with the theoretical expression?

Comparing the two equations, we can conclude that both represent a second degree function and, therefore, the trajectory of the projectile obeys to the horizontal launch equations.

In the equation $y = -0,788x^2 - 0,890x + 0,946$ we have: $\begin{cases} 0,866\ m = y_o \\ 0,106\ m^{-1} = \frac{g}{2v_0^2} \rightarrow v_o = 6,79\ m/s \end{cases}$

These values agree with those found in the experiment I:

$$\begin{cases} \textit{The measured value of } y_o \textit{ is } 0,867 \\ \quad \textit{The average value of the launch velocity is } 6,77\frac{m}{s} \end{cases}$$

# EXPERIMENT 06 – TRAJECTORY OF A PROJECTILE IN THE HORIZONTAL LAUNCH

**Objective: Verifying if a horizontally launch projectile describes a parabolic trajectory.**

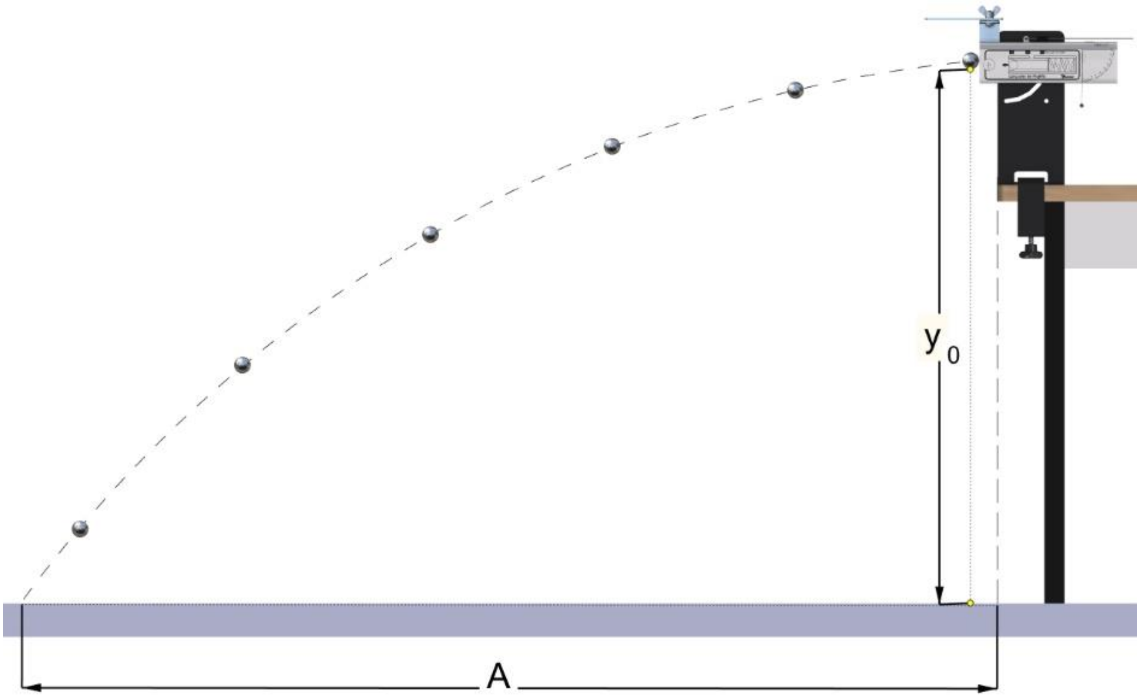
>>> ***Material Used*** >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

| Item | Code | Quant. | Unit | Description |
|---|---|---|---|---|
| 01 | 62002176 | 01 | Un. | CLAMP "C" |
| 02 | 62005751 | 01 | Un. | CANNON LAUNCHER HOLDER |
| 03 | 62002015 | 01 | Un. | CANNON (PROJECTILE LAUNCHER) |
| 05 | 62005120 | 01 | Un. | PLASTIC BALL Ø25MM |
| 06 | 62005177 | 01 | Un. | TUBE FOR CANNON COMPRESSION |
| 07 | 48005003 | 02 | Un. | BUTTERFLY NUT (CANNON'S FASTENER) |
| 08 | 50001004 | 02 | Un. | FLAT WASHER (CANNON'S FASTENER) |
| 09 | 62005317 | 01 | Un. | CANNON'S FASTENER |
| 10 | 62001023 | 01 | Un. | SENSORS/SPHERE SHOCK'S HOLDER |
| 13 | 03003011 | 01 | Un. | TAPE MEASURE 05M |
| 14 | 62005274 | 01 | Un. | PLUMB BOB WITH MAGNETIC FIXING |
| XX | 29003002 | 04 | Un. | BIG TRIPOD (*) |
| XX | 30002014 | 06 | Un. | PAIR OF ADAPTABLE STEMS 12,7MM X 400MM (*) |
| XX | 02005006 | 04 | Un. | LAB CLAMP WITH RING (*) |
| XX | 31003001 | 04 | Un. | METALLIC CLIP AZB-02 (*) |

(*) It does not accompany the product. It is **s**old separately.

>>> ***Experimental Procedures***>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

**1.** Assemble the launcher as shown.




**2.** Measure the launch height (yo) in relation to the floor. The measurement shall be made from the lower part of the ball indicated at the exit of the cannon, to the floor, according to the figure.

$y_0$= 0,947 m

**3.** Affix a paper sheet and mark the vertical alignment of the plumb with the center of the ball, as shown.

**4.** Place the plastic ball in the cannon and compress the spring to the first stage. Pull the trigger and observe where the ball touches the floor.

**5.** At the dropping point of the ball, affix a paper sheet and on it a carbon paper, marking the projectile's reach distance A.

**6.** Repeat the launch 5 times and measure the reached distance A by using the measuring tape.

Table 1

| N | Reach (m) |
|---|---|
| 1 | 1,485 |
| 2 | 1,490 |
| 3 | 1,496 |
| 4 | 1,515 |
| 5 | 1,512 |
| Average value | 1,497 |

>>> ***Analysis of Results and Conclusions*** >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

**7.** Determine the average reach distance.

$$A = \frac{1,485 + \cdots + 1,512}{5} = 1,497\ m$$

**8.** By combining equations:

$$\begin{cases} A = v_0 \cdot t \\ y = y_0 - \dfrac{g \cdot t^2}{2} \end{cases}$$

We obtain the equation that provides the launch velocity:

$$v_0 = A \cdot \sqrt{\frac{g}{2y_0}}$$

**9.** Calculate the projectile horizontal launch velocity $v_0$ by using the reach average value.

$$v_0 = 1,497 \cdot \sqrt{\frac{9,78}{2 \times 0,947}} = 3,402\ m/s$$

**10.** Draw on the sheet of paper placed on the floor a line that joins the point marked with the plumb bob and the point corresponding to the reach average value.

**11.** Mark from the origin along the line drawn on the paper the following positions:

$x_1$ = 0,30 m; $x_2$ = 0,60 m; $x_3$ = 0,90 m; $x_4$ = 1,20 m.

**12.** The vertical motion equation is:

$$y = y_o - \frac{g}{2}t^2; \quad (v_{0y} = 0)$$

By combining with horizontal motion equation, $x = v_0 \cdot t$, results the equation that relates the projectile positions y and x, y=f(x):

$$y = y_0 - \frac{g}{2v_0^2}x^2$$

**13.** Calculate the value of the vertical y position of the projectile for x positions marked on the sheet of paper and suggested in the table.

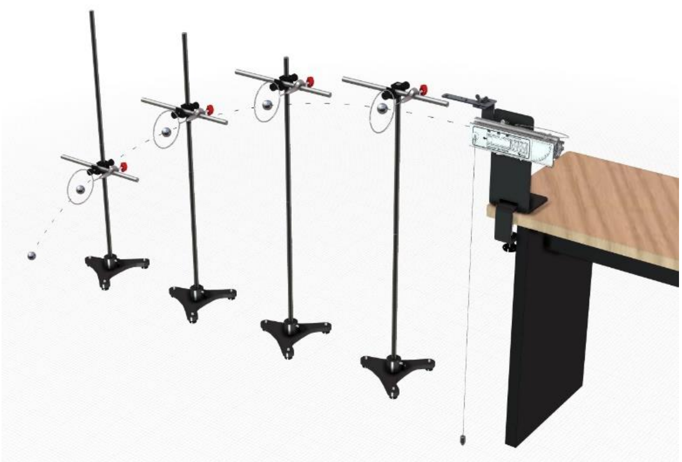
Table 2

| x (m) | $y_o$ (m) | $v_o$ (m/s) | y (m) |
|---|---|---|---|
| 0,300 | 0,947 | 3,402 | 0,909 |
| 0,600 | 0,947 | 3,402 | 0,795 |
| 0,900 | 0,947 | 3,402 | 0,605 |
| 1,200 | 0,947 | 3,402 | 0,339 |

$$y_1 = 0,947 - \frac{9,78}{2 \times 3,402^2} 0,300^2 = 0,909\ m \qquad y_2 = 0,947 - \frac{9,78}{2 \times 3,402^2} 0,600^2 = 0,795\ m$$

$$y_3 = 0,947 - \frac{9,78}{2 \times 3,402^2} 0,900^2 = 0,605\ m \qquad y_4 = 0,947 - \frac{9,78}{2 \times 3,402^2} 1,200 = 0,339\ m$$

**14.** Assemble four supports for the rings and position them in x positions suggested in table 2, as shown in the figure.



**15.** Prepare the projectile launch with the plastic ball in the first stage. Fire the launch and verify if the projectile has passed through the rings without touching them.

# EXPERIMENT 07 – CONSERVATION OF ENERGY

**Objective:**
**Verifying conservation of mechanical energy using the vertical launch.**

*>>> Material Used >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>*

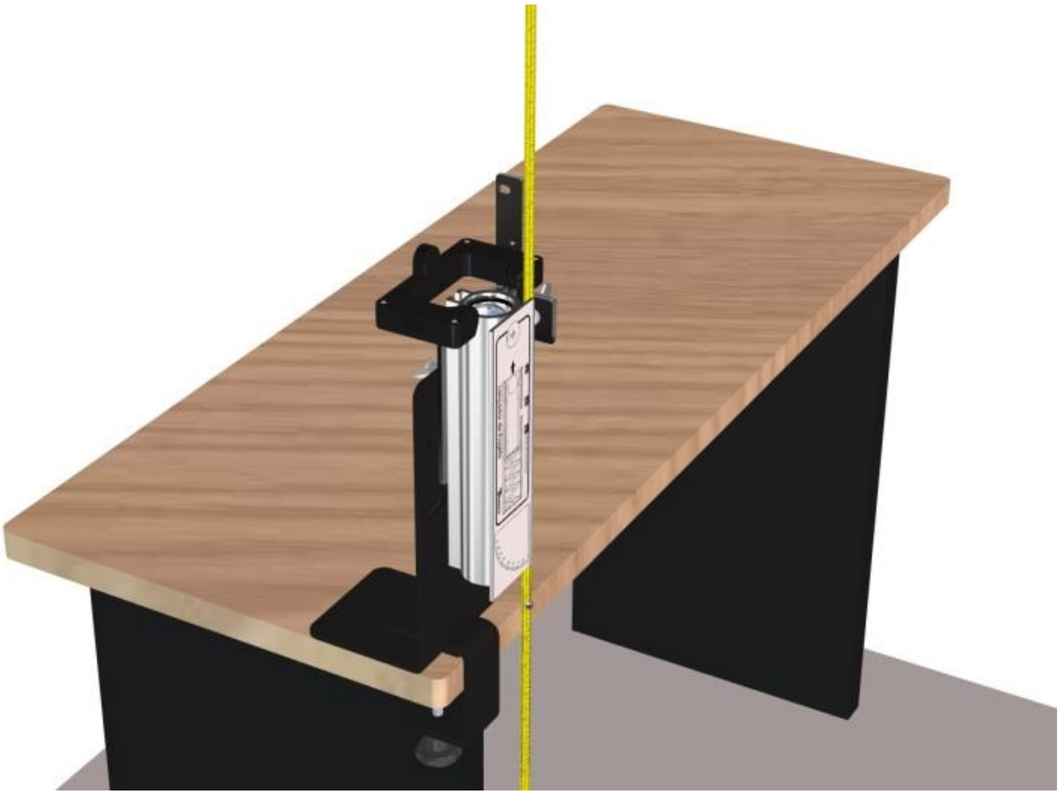
| *Item* | *Code* | *Quant.* | *Unit* | *Description* |
|---|---|---|---|---|
| *01* | *62002176* | *01* | *Un.* | *CLAMP "C"* |
| *02* | *62005751* | *01* | *Un.* | *CANNON LAUNCHER HOLDER* |
| *03* | *62002015* | *01* | *Un.* | *CANNON (PROJECTILE LAUNCHER)* |
| *04* | *62001074* | *01* | *Un.* | *STEEL BALL Ø25MM* |
| *06* | *62005177* | *01* | *Un.* | *TUBE FOR CANNON COMPRESSION* |
| *07* | *48005003* | *02* | *Un.* | *BUTTERFLY NUT (CANNON'S FASTENER)* |
| *08* | *50001004* | *02* | *Un.* | *FLAT WASHER (CANNON'S FASTENER)* |
| *09* | *62005317* | *01* | *Un.* | *CANNON'S FASTENER* |
| *10* | *62001023* | *01* | *Un.* | *SENSORS/SPHERE SHOCK'S HOLDER* |
| *13* | *03003011* | *01* | *Un.* | *TAPE MEASURE 05M* |
| *14* | *62005274* | *01* | *Un.* | *PLUMB BOB WITH MAGNETIC FIXING* |
| *XX* | *62001226* | *01* | *Un.* | *DIGITAL TIMER AZB-30 USB (*)* |
| *XX* | *62001201* | *01* | *Un.* | *PHOTOELECTRIC SENSOR PGS-D10 (*)* |

**(*) It does not accompany the product. It is sold separately.**

***>>> Experimental Procedures>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>***

Do not look directly at the cannon exit, as it may be charged!

**1.** Assemble the projectile launcher as shown.



**2.** Adjust the launch angle to 90º.

**3.** Measure the diameter (Ø) and the mass (m) of the steel ball.

**4.** Assemble a tripod with a stem on the table and fix an object (a ruler, for example) that serves as a reference to determine the vertical reach of the projectile, as shown in the figure.

**5.** Fix the sensor to measure the ball passage time, in the position shown in the figure.

**6.** Connect the timer and adjust it to use the function 2, with the screens shown below:



To select the desired measurement type, press the SETUP key for 2s to enter the configuration mode. In the configuration mode, use the START / RESET key to navigate among the configuration parameters. To change a selected parameter use the SETUP / MEM ↑ keys. After setting the function, press the FUNC key to save the selected parameters.

**7.** Position the steel ball in the second stage of the launcher and fire the cannon 5 times to determine the vertical reach value of the ball, adjusting the referential at each launch.

**8.** Note in table 1 the vertical reach and the time of passage values of the ball by the sensor.

Table 1

| N | y (m) | t(s) |
|---|---|---|
| 1 | 0,521 | 0,00785 |
| 2 | 0,521 | 0,00785 |
| 3 | 0,520 | 0,00781 |
| 4 | 0,519 | 0,00779 |
| 5 | 0,519 | 0,00781 |
| $y_{average}$ | 0,520 | 0,007822 |

**>>> *Analysis of Results and Conclusions* >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>**

**9.** Calculate the average value of the maximum height (y) reached by the ball.

$$y = \frac{0,525 + \cdots + 0,522}{5} = 0,520\ m$$

**10.** Calculate the average value of the passage time (t).

$$t = \frac{0,00785 + \cdots + 0,00781}{5} = 0,007822\ s$$

**11.** Use the ball diameter value and the average passage time and calculate the launch velocity modulus $v_0$.

$$v_0 = \frac{0,025}{0,007822} = 3,196\ m/s$$

**12.** Calculate the kinetic energy value in the initial position (at the time of launch).

$$K = \frac{m_{esf} \cdot v_0^2}{2} = \frac{0,064 \times 3,196^2}{2} = 0,327\ J$$

**13.** Calculate the value of the gravitational potential energy at the highest point of the path.

$$U = m_{esf} \cdot g \cdot y = 0,064 \times 9,78 \times 0,520 = 0,325\ J$$

**14.** Compare the values of mechanical energy in both procedures and justify the discrepancy found:

The difference between the two values is$\Delta E = K - U = 0,327 - 0,325 = 0,002\ J$

$$e\% = \frac{|K - U|}{\frac{K + U}{2}} \times 100\% = \frac{0,327 - 0,325}{\frac{0,327 + 0,325}{2}} \times 100\% = 0,43\%$$

This difference is due to deviations in the vertical reach measures, in obtaining the initial velocity and the loss by friction with the air.

**15.** Does the experiment confirm the principle of conservation of mechanical energy?

The values found, considering the error tolerance, can be considered equal, which confirms the validity of the principle.

# EXPERIMENT 08 – CONSERVATION OF THE LINEAR MOMENT

**Objective:**
**Verifying the conservation of linear momentum in elastic and inelastic collision.**

>>> ***Material Used*** >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

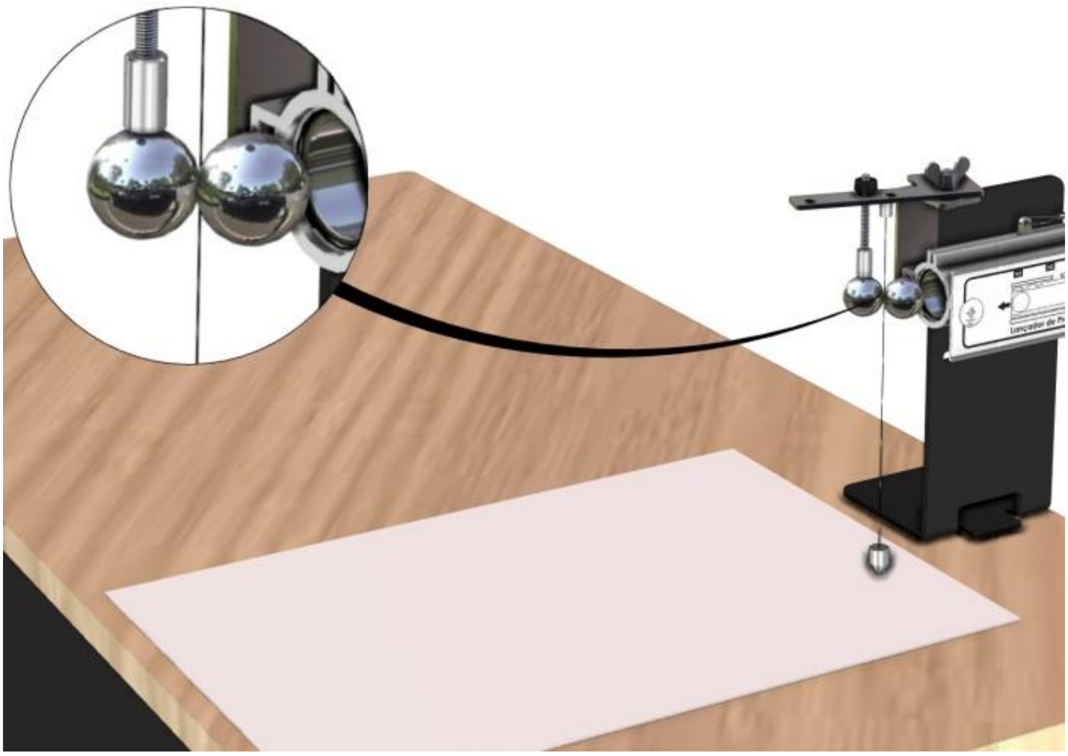
| *Item* | *Code* | *Quant.* | *Unit* | *Description* |
|---|---|---|---|---|
| *01* | *62002176* | *01* | *Un.* | *CLAMP "C"* |
| *02* | *62005751* | *01* | *Un.* | *CANNON LAUNCHER HOLDER* |
| *03* | *62002015* | *01* | *Un.* | *CANNON (PROJECTILE LAUNCHER)* |
| *04* | *62001074* | *02* | *Un.* | *STEEL BALL Ø25MM* |
| *06* | *62005177* | *01* | *Un.* | *TUBE FOR CANNON COMPRESSION* |
| *07* | *48005003* | *02* | *Un.* | *BUTTERFLY NUT (CANNON'S FASTENER)* |
| *08* | *50001004* | *02* | *Un.* | *FLAT WASHER (CANNON'S FASTENER)* |
| *09* | *62005317* | *01* | *Un.* | *CANNON'S FASTENER* |
| *10* | *62001023* | *01* | *Un.* | *SENSORS/SPHERE SHOCK'S HOLDER* |
| *11* | *62002056* | *01* | *Un.* | *BALL'S MAGNETIC FIXING (FOR COLLISION)* |
| *12* | *53003001* | *01* | *Un.* | *FASTENER FOR BALL'S MAGNETIC FIXING* |
| *13* | *03003011* | *01* | *Un.* | *TAPE MEASURE 05M* |
| *14* | *62005274* | *01* | *Un.* | *PLUMB BOB WITH MAGNETIC FIXING* |

## Part I – Elastic Oblique Collision

>>> ***Experimental Procedures***>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

Do not look directly at the cannon exit, as it may be charged!

**1.** Assemble the projectile launcher as shown.



**2.** Position the launcher for horizontal launch

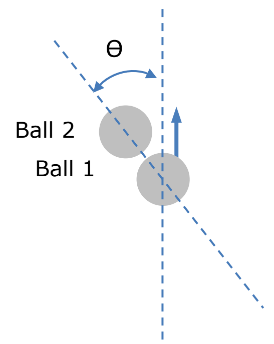
**3.** Measure the mass of each steel ball.

$m_1 = m_2$ = 0,064 kg.

**4.** Attach the magnetic fixing as shown previously.

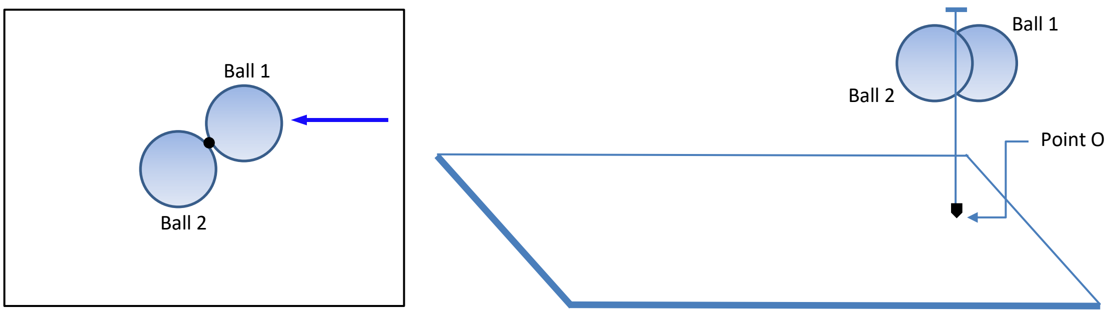
**5.** Fire the cannon once with the steel ball 1 in the second stage and adjust the position of the launcher to ensure that the ball falls on the sheet of paper covering the table.

**6.** Place the second steel ball (2) on the magnetic fixing and adjust its position very well for an oblique collision at an angle ϴ around 40 ° to 50 °. It is advisable that the collision occurs at least 3.0 cm from the launcher's mouth.




**7.** Fire launches that produce a collision between steel balls 1 and 2 and make adjustments to the launcher and the support of ball 2 positions that ensure that in these procedures (with and without collision) the balls fall on the table.

**8.** Use a plumb bob and determine on the sheet of paper placed on the table the point O where the collision occurs. This point will be considered as the origin of horizontal displacement.




**9.** Measure the launch height $h_o$ in relation to the table surface.

$h_o$ = 0,189 m

**10.** Remove the ball 2 from the magnetic support.

**11.** Cover the sheet of paper with carbon paper in the estimated position.

**12.** Fire five launches with ball 1 in the first stage and determine the point where the ball touches the table surface.

**13.** Measure the reach value in each launch, note them in table 1 and determine the average value $A_o$.

Tale 1

| N | Height $h_O$ (m) | Reach $A_o$ (m) |
|---|---|---|
| 1 | 0,189 | 0,525 |
| 2 | | 0,525 |
| 3 | | 0,532 |
| 4 | | 0,522 |
| 5 | | 0,525 |
| Average value | | 0,526 |

**14.** Place the ball 1 in the first stage of the launcher. Position the ball 2 on the magnetic fixing and fire the launcher. Observe where the balls touch the table surface after the collision. Place in these positions sheets of carbon paper.

**15.** Prepare a new launch for the two balls collision.

**16.** Fire the launcher and measure the two balls reaches $A_1$ and $A_2$ after the collision.

**17.** Repeat the launch process five times and determine the average position of each ball reach after the collision. Complete table 2

Table 2

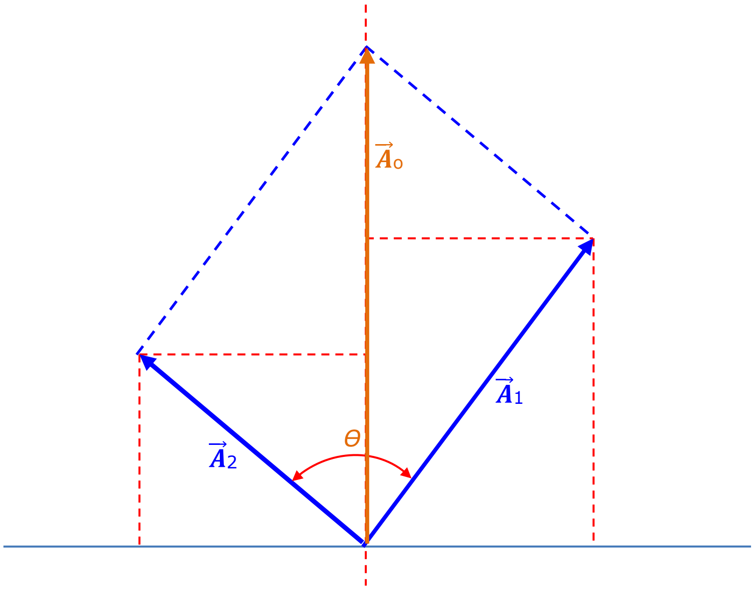
| *event* | *$A_1$ (m)* | *$A_2$ (m)* |
|---|---|---|
| *1* | *0,370* | *0,357* |
| *2* | *0,350* | *0,367* |
| *3* | *0,363* | *0,357* |
| *4* | *0,368* | *0,349* |
| *5* | *0,350* | *0,374* |
| *Average value* | *0,360* | *0,361* |

**>>> *Analysis of Results and Conclusions* >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>**

**18.** Draw on the sheet of paper from the point O, the lines corresponding to the average reaches $A_0$ of the ball 1 before the collision, A1 of the ball1 and A2 of the ball 2, after the collision.

**19.** Measure the angle Θ between the vectors corresponding to A1 and A2.

$\theta = 87^o$




**20.** Use the average value of table 1 and the appropriate equations to calculate the initial momentum modulus $p_o$ of the system:

$$v_o = A \cdot \sqrt{\frac{g}{2h_o}} \text{e} p_o = m_{esf} \cdot v_o$$

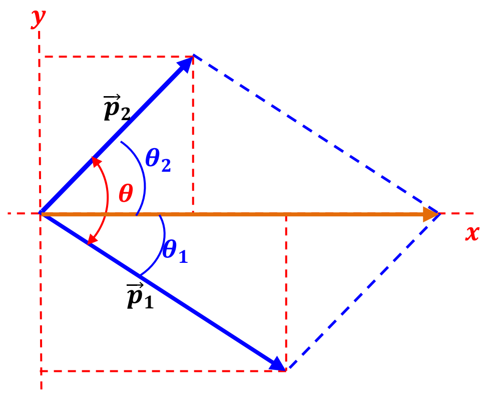
$$v_o = 0,526 \cdot \sqrt{\frac{9,78}{2 \cdot 0,189}} = 2,676\ m/s\ \text{e} p_o = 0,064 \cdot 2,604 \rightarrow p_o = 0,171\ kg \cdot m/s$$

**21.** Use the data in table 2 and calculate the values of linear momentum modules of each ball after the collision.

$$v_1 = 0,360 \cdot \sqrt{\frac{9,78}{2 \cdot 0,189}} = 1,831\ m/s \text{e} p_1 = 0,064 \cdot 1,831\ \rightarrow\ p_1 = 0,117 kg \cdot m/s$$

$$v_2 = 0,361 \cdot \sqrt{\frac{9,78}{2 \cdot 0,189}} = 1,836\ m/s \text{e} p_2 = 0,064 \cdot 1,836\ \rightarrow\ p_2 = 0,118\ kg \cdot m/s$$

**22.** Scale the vectors $\vec{p}_o$, $\vec{p}_1$ and $\vec{p}_2$.




**23.** Apply the cosine law and obtain the vector module $\vec{p}$, resulting from the vectors $\vec{p}_1$ and $\vec{p}_2$.

$$p^2 = 0,117^2 + 0,118^2 + 2 \cdot 0,117 \cdot 0,118 \cdot cos87^o \rightarrow p = 0,170\ kg \cdot m/s$$

**24.** Compare the values of the linear momentum modules just before and immediately after the collision. Has the linear momentum of the system been conserved?

Table 3

| *Initial linear momentum $p_o$ (kg.m/s)* | *Final linear momentum (kg.m/s)* | *Percent error* |
|---|---|---|
| *0,171* | *0,170* | *0,6%* |

**25.** Take the direction x of reference as the direction of the ball's reach before the collision and trace the pair of axes (x, y). Measure the angles that each vector forms with the x-axis.

$\theta_1 = -39^o$ and $\theta_2 = 49^o$

**26.** Obtain the orthogonal components of the linear moments after the collision:

Table 4

| | *Module* | *Angle with x-axis* | *Component x $p_{ox}$* | *Component y $p_{oy}$* |
|---|---|---|---|---|
| $p_o$ | *0,171* | *0* | *0,171* | *0* |
| $p_1$ | *0,117* | *-43º* | *0,0858* | *-0,080* |
| $p_2$ | *0,118* | *44⁰* | *0,0845* | *0,082* |
| $p$ | | *0* | *0,1703* | *-0,001* |

**27.** Can we consider that in the direction x the linear momentum has been conserved? And in the direction y?

Initially the linear moment has the direction x and its modulus is 0.171kg.m/s. The components x sum of the final linear momentum is 0.170 kg.m/s. The difference can be neglected and the linear momentum in the direction x is conserved.

The initial linear momentum has no component y and the components y sum of the final linear momentum -0.001 kg.m / s can be neglected, and it can be considered that the linear momentum was conserved in the Direction y.

**28.** Was there conservation of kinetic energy? Can collision be considered as an elastic collision? Justify.

$$K_{inicial} = \frac{0,064 \times 2,676^{\,2}}{2} = 0,229\, J K_1 = \frac{0,064 \times 1,831^2}{2} = 0,107\, J K_2 = \frac{0,064 \times 1,836^2}{2} = 0,108\, J$$

Table 5

| Initial kinetic energy (J) | Final kinetic energy (J) | | | |
|---|---|---|---|---|
| | Ball 1 | Ball 2 | Sum | e% |
| 0,229 | 0,107 | 0,108 | 0,215 | 6,3% |

The values of the kinetic energies, initial and final, are approximately equal. The difference in the values of 0.014 J indicates that the collision should be considered partially elastic.

## Part II – Inelastic Oblique Collision

**>>> *Experimental Procedures*>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>**

Do not look directly at the cannon exit, as it may be charged!

**1.** Use the same assembly of the first part with steel balls.

**2.** Consider the data from the table obtained with the ball 1 launch without collision.

Table 1

| N | Launch height $h_O$ (m) | Reach $A_o$ (m) |
|---|---|---|
| 1 | | 0,525 |
| 2 | | 0,525 |
| 3 | 0,189 | 0,532 |
| 4 | | 0,522 |
| 5 | | 0,525 |
| Average value | | 0,526 |

**3.** Adopt as axis x the launch direction without collision.

**4.** Wrap the ball 2 with one lap of sticky tape and repeat the collision launch procedures. In this case it is advisable to Fire only one launch, since the replacement of the ball 2 would hardly occur in the same positioning conditions

**5.** Lay on the table a sheet of paper where the balls fall after the collision.

**6.** Repeat the procedures for marking the point of origin of the horizontal displacements of the balls.

**7.** Fire some previous launches and observe the points where the balls touch the table. Put sheets of carbon paper at these points.

**8.** Fire the cannon and score the points obtained from the two balls reaches.

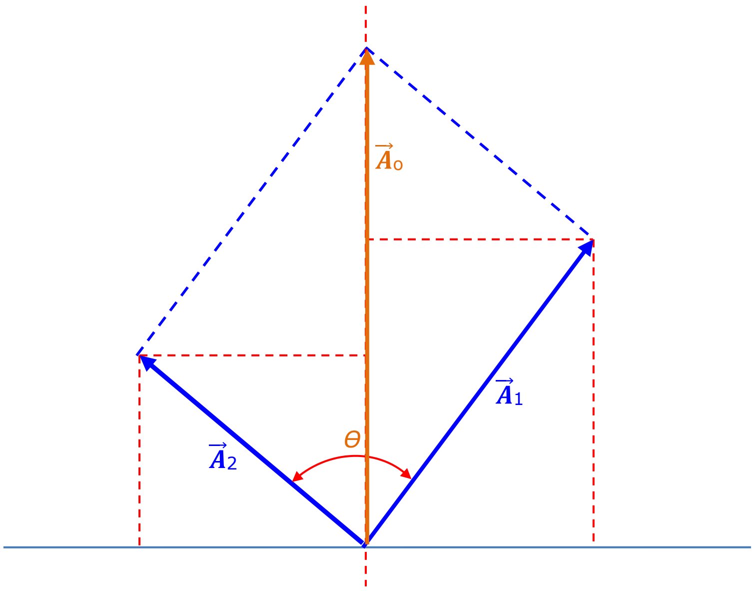
| | Reach (m) | Angle with axis x |
|---|---|---|
| Ball 1 | 0,340 | 23,5º |
| Ball 2 | 0,250 | 36,5º |

**>>> *Analysis of Results and Conclusions* >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>**

**9.** Calculate the value of the linear momentum module of the system immediately before the collision.

$$v_o = 0,526 \cdot \sqrt{\frac{9,78}{2\cdot 0,189}} = 2,676\ m/s \text{ e} p_o = 0,064 \cdot 2,604 \rightarrow p_o = 0,171\ kg \cdot m/s$$

**10.** Draw on the sheet of paper from the point O, the lines corresponding to the average reaches A0 of the ball 1 before the collision, A1 of the ball1 and A2 of the ball 2, after the collision.




**11.** Measure the angle that each vector forms with the vector direction obtained without the collision.

**12.** Measure the angle Θ between the vectors corresponding to A1 and A2.

ϴ = 60º

**13.** Calculate the value of the velocity and momentum modules of each ball after the collision. The launch height $h_0$ is the same as the first part. ($h_o$ =0,189 m).

$$v = A \cdot \sqrt{\frac{g}{2h_o}} \text{ e } p_o = m_{esf} \cdot v_o$$

$$v_1 = 0,340 \cdot \sqrt{\frac{9,78}{2 \cdot 0,189}} = 1,729 m/s \text{ e } p_1 = 0,064 \cdot 1,729 \rightarrow p_1 = 0,1107 kg \cdot m/s$$

$$v_2 = 0,250 \cdot \sqrt{\frac{9,78}{2 \cdot 0,189}} = 1,272\ m/s \text{ e } p_2 = 0,064 \cdot 1,272 \rightarrow p_2 = 0,0827\ kg \cdot m/s$$

**14.** Use the cosine law and calculate the value of the linear momentum module after the collision.

$$p^2 = 0,1107^2 + 0,0827^2 + 2 \cdot 0,1107 \cdot 0,0827 \cdot cos60^0$$
$$p = 0,168\ kg.m/s$$

**15.** Was there conservation of the linear momentum of the system in the inelastic collision?

By comparing the values found for the initial and final linear momentum:

$$\begin{cases} p_o = 0,171\ kg.m/s \\ p = 0,168\ kg.m/s \end{cases}$$

As they present a difference much smaller than the error tolerance of 5%, we can affirm that the linear momentum of the system was conserved.

**16.** Calculate the initial and final kinetic energy of the system. Was there energy conservation in the collision?

$$\begin{cases} \boldsymbol{K_{initial} = \frac{0,064 \cdot 2,676^2}{2} = 0,229\,J} \\ \boldsymbol{K_{final} = \frac{0,064 \cdot 1,729^2}{2} + \frac{0,065 \cdot 1,272^2}{2} = 0,148\,j} \end{cases}$$

The results show that there was no energy conservation and therefore, it is an inelastic collision.

# Part III – Frontal Elastic Collision

**>>> *Experimental Procedures*>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>**

Do not look directly at the cannon exit, as it may be charged!

**1.** Use the same assembly as in the first part.

**2.** Consider the data of table 1 obtained with the ball 1 launch without collision.

Table 1

| N | Launch height $h_0$ (m) | Reach $A_0$ (m) |
|---|---|---|
| 1 | 0,189 | 0,525 |
| 2 | | 0,525 |
| 3 | | 0,532 |
| 4 | | 0,522 |
| 5 | | 0,525 |
| Average value | | 0,526 |

**3.** Adopt as x-axis the launch direction without collision.

**4.** Adjust the position of the ball 2 support so the centers of the two balls are on the same axis providing a frontal collision.

**5.** Lay on the table a sheet of paper to accommodate the balls after the collision.

**6.** Fire the cannon a couple of times and adjust the position of ball 2 until a really frontal collision (without lateral deviation of the balls after the collision).

**7.** Cover the sheet with carbon paper in the regions where the balls touch the table.

**8.** Trigger the launcher and mark the reaches' points A1 and A2 of the two balls.

**>>> *Analysis of Results and Conclusions* >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>**

**9.** Draw the lines corresponding to the scales, measure and note the value of the reaches modules $A_1$ and $A_2$ of each ball.

**10.** Measure the angles Θ1 and Θ2 which A1 and A2 form with the x-axis and the angle Θ between A1 and A2.

If the balls position is correct, the lines corresponding to the reaches must coincide with the x-axis direction, and therefore $\theta_1 = \theta_2 = \theta$=0.

Table 2

| | Reach (m) | Angle with x-axis |
|---|---|---|
| Ball 1 | 0,015 | 0 |
| Ball 2 | 0,500 | 0 |

**11.** Calculate the value of the velocity and momentum modules of each ball after the collision. The launch height $h_0$ is the same as the first part. ($h_o$ =0,189 m).

$$v_o = A \cdot \sqrt{\frac{g}{2h_o}} \text{e} p_o = m_{esf} \cdot v_o$$

$$v_1 = 0,015 \cdot \sqrt{\frac{9,78}{2 \cdot 0,189}} = 0,076 m/s \text{e} p_1 = 0,064 \cdot 0,076 \rightarrow p_1 = 0,005\ kg \cdot m/s$$

$$v_2 = 0,500 \cdot \sqrt{\frac{9,78}{2 \cdot 0,189}} = 2,543\ m/s \text{e} p_2 = 0,064 \cdot 2,543 \rightarrow p_2 = 0,165\ kg \cdot m/s$$

**12.** Obtain the linear momentum module after the collision.

Since the two vectors are collinear (coincident with the x-axis) the final linear momentum module will be:

$$p = p_1 + p_2 = 0,005 + 0,165 \rightarrow p = 0,170\ kg.m/s$$

**13.** Was there conservation of the linear momentum of the system in the inelastic collision?

By comparing the values found for the initial and final linear momentum:

$$\begin{cases} p_o = 0,171\ kg.m/s \\ p = 0,170\ kg.m/s \end{cases}$$

As they present a difference much smaller than the error tolerance of 5%, we can affirm that the linear momentum of the system was conserved.

**14.** Calculate the initial and final kinetic energy of the system. Was there energy conservation in the collision?

$$\begin{cases} K_{initial} = \dfrac{0,064 \cdot 2,676^2}{2} = 0,229\ J \\ K_{final} = \dfrac{0,064 \cdot 0,076^2}{2} + \dfrac{0,064 \cdot 2,543^2}{2} = 0,207\ J \end{cases}$$

The results show that the energy was partially conserved and, therefore, it is a partially elastic collision.

## Part IV – Frontal Inelastic Collision

>>> ***Experimental Procedures***>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

Do not look directly at the cannon exit, as it may be charged!

**1.** Use the same assembly as in the first part.

**2.** Consider the data of table 1 obtained with the ball 1 launch without collision.

Table 1

| N | Launch height $h_O$ (m) | Reach $A_o$ (m) |
|---|---|---|
| 1 | 0,189 | 0,525 |
| 2 | | 0,525 |
| 3 | | 0,532 |
| 4 | | 0,522 |
| 5 | | 0,525 |
| Average value | | 0,526 |

**3.** Adopt as x-axis the launch direction without collision.

**4.** Wrap ball 2 with one lap of sticky tape and repeat collision launch procedures. In this case it is advisable to fire only one time, since the replacement of the ball 2 would hardly occur in the same positioning conditions.

**5.** Adjust the position of the ball 2 support so the centers of the two balls are on the same axis providing a frontal collision.

**6.** Lay on the table a sheet of paper to accommodate the balls after the collision.

**7.** Fire a couple of times and adjust the position of ball 2 until a really frontal collision (without lateral deviation of the balls after the collision).

**8.** Cover the sheet with carbon paper in the regions where the balls touch the table.

**9.** Fire the launcher and mark the reaches' points A1 and A2 of the two balls.

>>> ***Analysis of Results and Conclusions*** >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

**10.** Draw the lines corresponding to the scales, measure and note the value of the reaches modules A1 and A2 of each ball.

**11.** Measure the angles $\Theta_1$ and $\Theta_2$ which $A_1$ and $A_2$ form with the x-axis and the angle $\Theta$ between $A_1$ and $A_2$.

If the balls position is correct, the lines corresponding to the reaches must coincide with the x-axis direction, and therefore $\theta_1 = \theta_2 = \theta = 0$.

Table 2

| | Reach (m) | Angle with x-axis |
|---|---|---|
| Ball 1 | 0,178 | 0 |
| Ball 2 | 0,332 | 0 |

**12.** Calculate the value of the velocity and momentum modules of each ball after the collision. The launch height h₀ is the same as the first part. ($h_0$ =0,189 m).

$$v_o = A \cdot \sqrt{\frac{g}{2h_o}} \text{e} p_o = m_{esf} \cdot v_o$$

$$v_1 = 0,178 \cdot \sqrt{\frac{9,78}{2 \cdot 0,189}} = 0,905 m/s \text{e} p_1 = 0,064 \cdot 0,905 \rightarrow p_1 = 0,058\, kg \cdot m/s$$

$$v_2 = 0,332 \cdot \sqrt{\frac{9,78}{2 \cdot 0,189}} = 1,689\, m/s \text{e} p_2 = 0,065 \cdot 1,689 \rightarrow p_2 = 0,110\, kg \cdot m/s$$

**13.** Obtain the linear momentum module after the collision.

Since the two vectors are collinear (coincident with the x-axis) the final linear momentum module will be:

$$p = p_1 + p_2 = 0,058 + 0,110 \rightarrow p = 0,168\, kg.m/s$$

**14.** Was there conservation of the linear momentum of the system in the inelastic collision?

By comparing the values found for the initial and final linear momentum:

$$\begin{cases} p_o = 0,171\, kg.m/s \\ p = 0,168\, kg.m/s \end{cases}$$

As they present a difference much smaller than the error tolerance of 5%, we can affirm that the linear momentum of the system was conserved.

**15.** Calculate the initial and final kinetic energy of the system. Was there energy conservation in the collision?

$$\begin{cases} K_{inicial} = \dfrac{0,064 \cdot 2,676^2}{2} = 0,229\, J \\ K_{final} = \dfrac{0,064 \cdot 0,905^2}{2} + \dfrac{0,065 \cdot 1,689^2}{2} = 0,119 J \end{cases}$$

The results show that there was a reasonable loss of energy (ΔK = 0.110 J) and, therefore, it is an inelastic collision.

# EXPERIMENT 09 – LAUNCH ANGLE THAT MAXIMIZES THE HEIGHT OF A PROJECTILE

**Objective:**
**- Finding the launch angle that maximizes the height reached by projectile launched from a fixed distance from a bulkhead.**

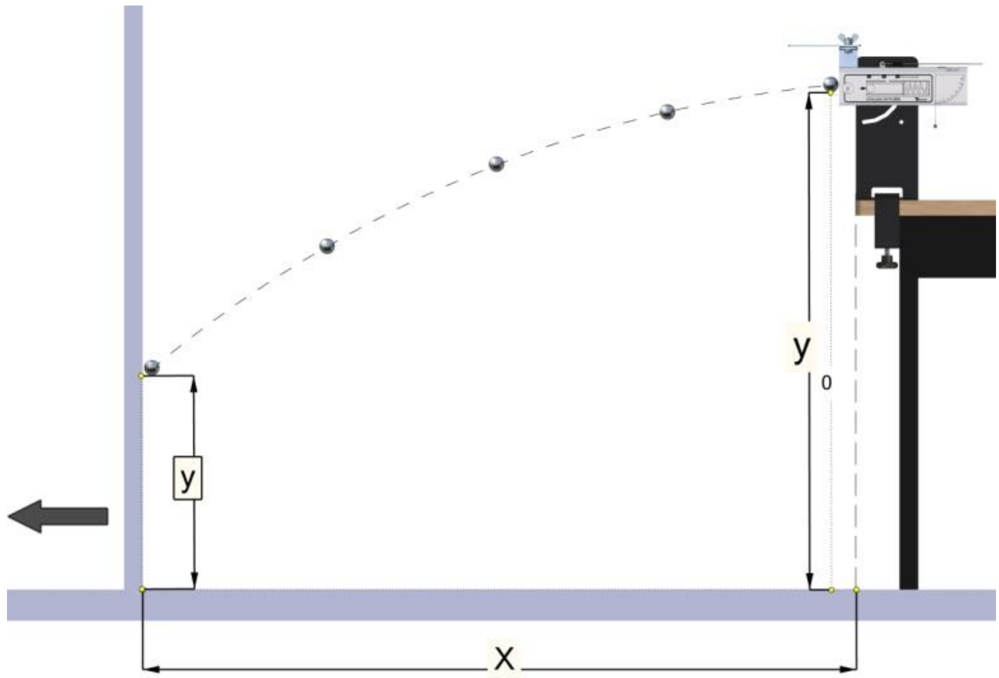
**>>> *Material Used* >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>**

| Item | Code | Quant. | Unit | Description |
|---|---|---|---|---|
| 01 | 62002176 | 01 | Un. | CLAMP "C" |
| 02 | 62005751 | 01 | Un. | CANNON LAUNCHER HOLDER |
| 03 | 62002015 | 01 | Un. | CANNON (PROJECTILE LAUNCHER) |
| 05 | 62005120 | 01 | Un. | PLASTIC BALL Ø25MM |
| 06 | 62005177 | 01 | Un. | TUBE FOR CANNON COMPRESSION |
| 07 | 48005003 | 02 | Un. | BUTTERFLY NUT (CANNON'S FASTENER) |
| 08 | 50001004 | 02 | Un. | FLAT WASHER (CANNON'S FASTENER) |
| 09 | 62005317 | 01 | Un. | CANNON'S FASTENER |
| 13 | 03003011 | 01 | Un. | TAPE MEASURE 05M |
| 14 | 62005274 | 01 | Un. | PLUMB BOB WITH MAGNETIC FIXING |

**>>> *Experimental Procedures*>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>**

Do not look directly at the cannon exit, as it may be charged!

**1.** Assemble the launcher as shown.




**2.** Position the table facing a wall so the launcher's mouth is about 1.60 m away. Use a plumb bob to determine the initial launch position on the floor and measure the distance (x) from the wall to the launcher's mouth and the vertical launch position ($y_o$).

$x$=1,60 m $y_o$ = 0,923 m

**3.** Fire the cannon and determine where to place a 2 sheet of paper, displayed vertically.

**4.** Adjust the launch angle to 10º and insert the plastic ball into the second stage.

**5.** Fire launches to locate the point where the ball touches the wall. Attach at this point a sheet of carbon paper.

**6.** Fire the launch and measure the height (from the floor) reached by the projectile.

**7.** Launch at the angles suggested in table 1 and note the respective heights reached.

Table 1

| *Angle θ (degree)* | *10* | *15* | *20* | *25* | *30* | *35* | *40* | *45* | *50* | *55* | *60* | *65* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *Height y (m)* | *0,45* | *0,56* | *0,67* | *0,77* | *0,86* | *0,94* | *1,00* | *1,05* | *1,05* | *0,97* | *0,74* | *0,23* |

$$y = -\frac{g}{2 \cdot v_o^2 \cdot cos^2\theta} \cdot x^2 + (tg\theta) \cdot x + y_o$$

>>> ***Analysis of Results and Conclusions*** >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

**8.** Note in table 2 the launch velocity value obtained in experiment 1.

**9.** Use the initial velocity obtained in experiment 1.

$V_o$ = 4,12 m/s

**10.** By using the initial velocity of step 1 and the distance from the wall to the launcher, calculate the angle that provides the maximum height.

By applying the concept of maximum and minimum of a function we obtain the equation of the coordinate of the point where y = f (θ) is maximum:

$$tg\theta_{max} = \frac{v_0^2}{gx}$$

$$tg\theta_{max} = \frac{4,12^2}{9,78 \times 1,60} \rightarrow tg\theta_{max} = 1,08 \rightarrow \theta_{max} = 47,3^0$$

**11.** Draw the graph of the vertical position versus the launch angle of table 1.

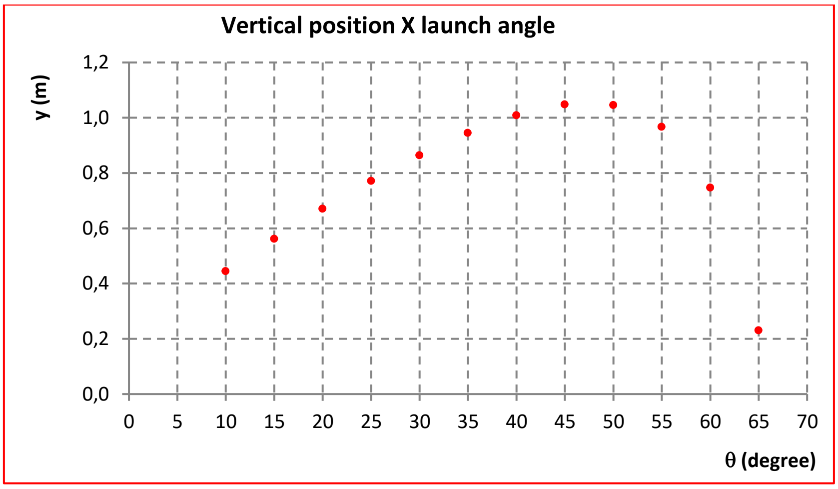


Table 2

| | *Valor* |
|---|---|
| *Angle for maximum height-measured* | *(*)* |
| *Maximum height* | *1,683 m* |
| *Horizontal wall distance* | *1,600 m* |
| *Launch height* | *0,923 m* |
| *Initial velocity calculated* | *4,12 m/s* |
| *Angle for maximum height-calculated* | *47,3º* |
| *Percentage difference among angles* | *2,2%* |

**12.** Does the value found analytically agree with the values obtained experimentally? Justify.

As the graph shows, the value of the launch angle, for $y_{max}$ is between 45º and 50º. The analytical value was $\theta_{max}$ = 47.3º which shows consistency between the two values.

**13.** For the angle that gives the maximum height when the ball hits the wall, has it reached the peak of the trajectory?

The velocity equation of vertical motion is:

$$v_y = v_o \cdot (sen\theta) - gt$$

At the point where the ball reaches the maximum height $v_y = 0$, then the climb time will be:

$$t_{subida} = \frac{v_0 \cdot sen\theta_{max}}{g} e t_{subida} = \frac{4{,}12 \cdot sen47{,}3^0}{9{,}78} = 0{,}310\ s$$

For this time interval, we can calculate the horizontal displacement of the projectile when it reaches the maximum height of the trajectory.

$$x = v_0 \cdot (cos\theta) \cdot t_{subida} = 4{,}12 \cdot (\cos 47{,}3^0) \cdot 0{,}310 = 0{,}87\ m$$

As the distance of the wall is 1.60 m, the projectile has already reached the peak of the trajectory by touching the wall.

**14.** How far from the wall will the height be maximized for a launch at a 45° angle? What would be the maximum height in this case?

$$tg\theta_{max} = \frac{v_0^2}{gx} \rightarrow x = \frac{v_0^2}{g \cdot tg\theta_{max}} = \frac{4{,}12^2}{9{,}78 \cdot tg45^0} = 1{,}74\ m$$

By using the equation for height y that the projectile touches the wall, we have:

$$y(x = 1{,}74m) = 0{,}923 + 1{,}74 \cdot tg45^0 - \frac{9{,}78 \cdot 1{,}74^2}{2 \cdot 4{,}12^2 . cos^2 45^0} \rightarrow y(x = 1{,}74m;\ \theta = 45^0) = 0{,}92\ m$$

**15.** Launch to x equal to the value found for the 45° angle and measure the y value (x; 45°). Compare with calculated value.

$$\begin{cases} y_{medido} = 0{,}90\ m \\ y_{calculado} = 0{,}92\ m \end{cases} e\% = \frac{0{,}92 - 0{,}90}{\frac{0{,}92 + 0{,}90}{2}} \times 100\% = 2{,}2\%$$

# EXPERIMENT 10 – BALLISTIC PENDULUM - APPROXIMATE METHOD

**Objectives:**
**- Using the conservation of linear momentum and conservation of mechanical energy in a ballistic pendulum to find the launch velocity of a projectile.**
**- Comparing the launch velocity found with the velocity obtained in a horizontal launch by using different processes.**

>>> ***Material Used*** >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

*PROJECTILE LAUNCHER*

| *Item* | *Code* | *Quant.* | *Unit* | *Description* |
|---|---|---|---|---|
| *01* | *62002176* | *01* | *Un.* | *CLAMP "C"* |
| *02* | *62005751* | *01* | *Un.* | *CANNON LAUNCHER HOLDER* |
| *03* | *62002015* | *01* | *Un.* | *CANNON (PROJECTILE LAUNCHER)* |
| *04* | *62001074* | *01* | *Un.* | *STEEL BALL Ø25MM* |
| *06* | *62005177* | *01* | *Un.* | *TUBE FOR CANNON COMPRESSION* |
| *07* | *48005003* | *02* | *Un.* | *BUTTERFLY NUT (CANNON'S FASTENER)* |
| *08* | *50001004* | *02* | *Un.* | *FLAT WASHER (CANNON'S FASTENER)* |
| *09* | *62005317* | *01* | *Un.* | *CANNON'S FASTENER* |
| *13* | *03003011* | *01* | *Un.* | *TAPE MEASURE 05M* |
| *14* | *62005274* | *01* | *Un.* | *PLUMB BOB WITH MAGNETIC FIXING* |

*Ballistic Pendulum Accessories(sold separately)*

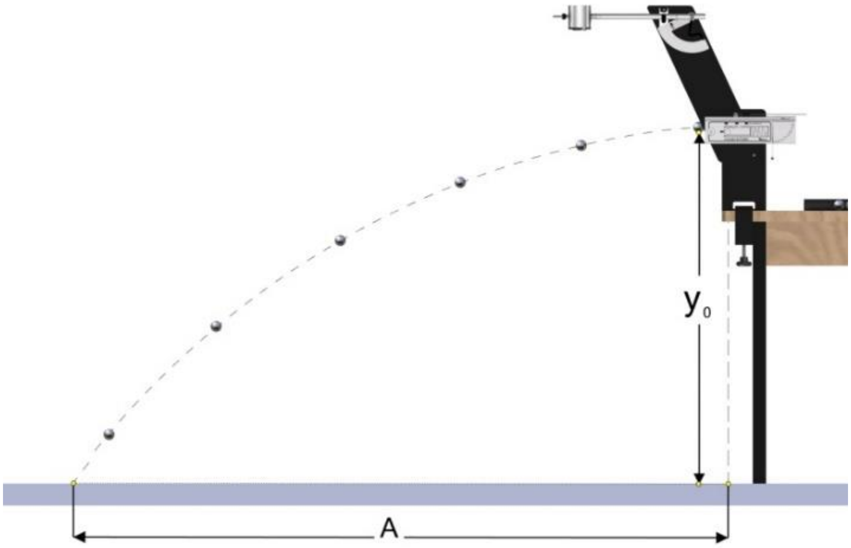
| *Item* | *Code* | *Quant.* | *Unit* | *Description* |
|---|---|---|---|---|
| *01* | *62005611* | *02* | *Un.* | *50G CYLINDER MASS* |
| *03* | *53003001* | *01* | *Un.* | *PENDULUM'S TOWER THUMB SCREW'S NUT* |
| *04* | *53001009* | *01* | *Un.* | *PENDULUM'S THUMB SCREW* |
| *04* | *53004002* | *01* | *Un.* | *METAL HANDLE M3X10* |
| *05* | *62002055* | *01* | *Un.* | *BALLISTIC PENDULUM* |
| *06* | *62002182* | *01* | *Un.* | *PENDULUM'S TOWER* |

## Part I - Obtaining the horizontal launch velocity by using the reach measure.

>>> ***Experimental Procedures***>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

Do not look directly at the cannon exit, as it may be charged!

**1.** Assemble the ballistic pendulum as shown.




2. Measure the launch height (yo) in relation to the floor. The measurement shall be made from the lower part of the ball indicated at the exit of the cannon, to the floor, according to the figure.

   $y_0$= 0,923 m

3. With a plumb bob, mark on a sheet of A4 paper pasted with adhesive tape on the floor, the post position (origin of horizontal displacement). The plumb bob must match the vertical passing through the center of the ball.

4. Place the steel ball in the projectile launcher and compress the spring to the third stage. Pull the trigger and observe where the ball touches the floor.

5. At the dropping point of the ball, affix a paper and on it a sheet of carbon paper for marking the projectile reach distance A.

**6.** Repeat the launch 5 times and measure the reach distance A.

Table 1

| N | Reach (m) |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| Average value | |

**7.** Use the horizontal launch equations and calculate the launch velocity:

# Part II - Obtaining launch velocity by using the conservation of linear momentum in a ballistic pendulum

**>>> *Experimental Procedures*>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>**

Do not look directly at the cannon exit, as it may be charged!

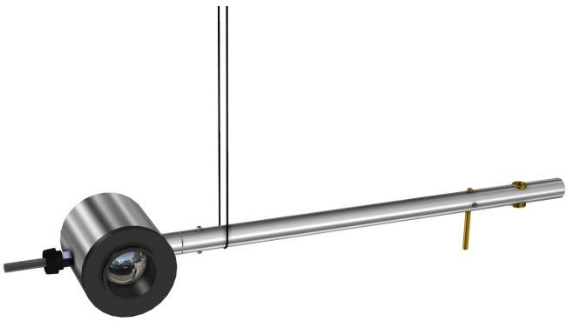
**1.** Use the 1st part launcher assembly at an end of the table.

**2.** Measure the mass m of the metal ball and the mass M of the set (ballistic pendulum + ball).

m=0,064 kg e M = 0,294 kg

**3.** Insert the ball into the pendulum receiver and use a line to find the center of mass position of the set. Slide the line holding the pendulum until it remains in horizontal balance.

**4.** Measure the distance R from the center of mass position to the axis of rotation.




R = 0,204 m

**5.** Attach the pendulum to the launcher support and make the necessary adjustments.

**6.** For the correct positioning of the pendulum the following steps are suggested:

- Keep the pendulum free in vertical.
- Carefully approach the cannon until it softly engages the mouth of the pendulum trimmer.
- Tighten the cylinder locking screws.

With these procedures the ball, when launched, will always be correctly picked up by the pendulum receiver.

**7.** Place the steel ball in the projectile launcher and compress the spring to the second stage.

**8.** Adjust the pendulum position so the axis of trimmer fits correctly into the launcher's mouth.

**9.** Adjusting the zero of the angle marker.

**10.** Release the trigger for launch and note the angle value reached by the pendulum.

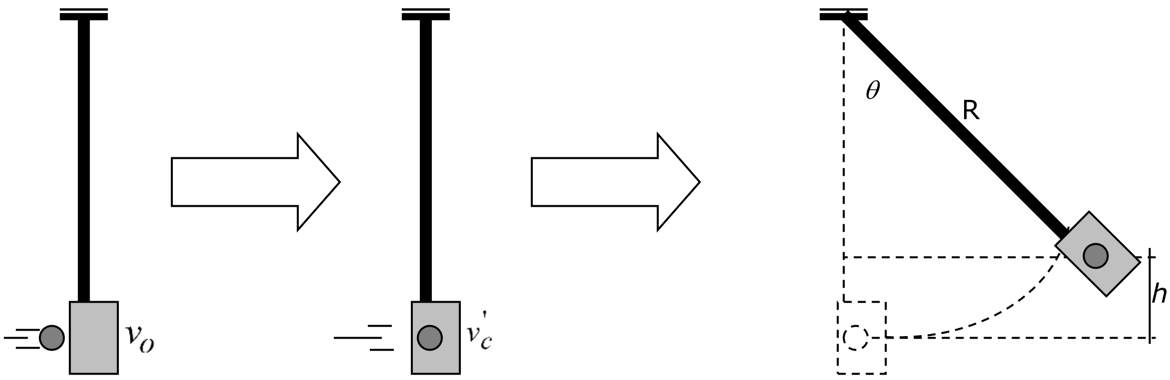
**11.** Write down the measurements of angles of the pendulum.

**12.** Choose the most repeatable measure and adjust the indicator to about four degrees less than that angle.

**13.** Repeat the launch three times and calculate the average value of the angle.

Table 1

| $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_{average}$ |
|---|---|---|---|
| 45,0º | 44,5º | 44,5º | 44,70º |




(a) (b) (c)

**>>>Theoretical Foundations>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>**

The height *h* reached by the center of mass of the pendulum is calculated by:

$$\mathbf{h = R(1 - cos\theta)}$$

Considering that the linear momentum of the system is conserved both in an elastic collision and in an inelastic collision, we have:

$$p_{before\ collision} = p_{after\ collision} \rightarrow mv_0 = M \cdot v \quad (1)$$

After the collision the ballistic pendulum moves and its center of mass reaches a height h:
As mechanical energy is conserved in this movement, we have:

$$K_{after\ collision} = U_{max\ hieght} \rightarrow \frac{Mv^2}{2} = MgR(1 - cos\theta) \rightarrow v = \sqrt{2 \cdot g \cdot R(1 - cos\theta)} \quad (2)$$

$$v = \sqrt{2 \times 9,78 \times 0,204 \times (1 - cos44,7)} = 1,07\ m/s$$

By combining (1) and (2), we obtain the value of the launch velocity before the pendulum collision ($v_o$):

$$v_o = \frac{M}{m}\sqrt{2 \cdot g \cdot R(1 - cos\theta)}$$

**>>> *Analysis of Results and Conclusions* >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>**

**14.** Use the deduced equation to calculate the ball launch velocity value.

$$v_0 = \frac{0,294}{0,064}\sqrt{2 \cdot 9,78 \cdot 0,204(1 - cos44,7^0)}$$

$$v_0 = 4,93\ m/s$$

**15.** Compare the result found with the average velocity of the ball obtained in the horizontal launch in the first part.

In the first part it was found the ball velocity just before the collision $v_{01}$.

$$v_{01} = 4,46\ m/s$$

**16.** Calculate the percentage difference between the two results found: $d\% = \frac{|A-B|}{\frac{A+B}{2}} \times 100\%$

$$d\% = \frac{|4,46 - 4,93|}{\frac{4,46 + 4,93}{2}} \times 100\% = 10\%$$

**17.** Does the percentage difference obtained confirm the validity of the conservation principle of linear momentum? Justify.

As you know, regardless of the type of collision, the linear momentum must be conserved. However, in this experiment it was considered that the pendulum-ball system behaves as a specific mass located in the center of mass of the set and the geometric distribution of the pendulum and the ball influence the equations used. In addition, the procedure used did not consider the rotational inertia existing in the studied phenomenon and, therefore, the result does not present a good precision. Therefore, this method is called the "approximate method".

**18.** What can be concluded about the energy conservation in the collision?

There are two moments when you consider the energy of the system:

**a.** The kinetic energy of the ball immediately before the collision with the pendulum:

$$K_{antes} = \frac{m_{esf} \cdot v_o^2}{2} = \frac{0,064 \times 4,46^2}{2} = 0,637\ J$$

**b.** The kinetic energy of the set immediately after the collision:

$$K_{depois} = \frac{M \cdot v_{conj}^2}{2} = \frac{0,294 \times 1,07^2}{2} = 0,168\ J$$

$$\Delta K = 0,168 - 0,637 = -0,469\ J$$

There was a loss of 74% of energy which shows that there was no energy conservation and therefore, it is an inelastic collision. This was already expected, as the bodies remained united after the collision.

# EXPERIMENT 11 – BALLISTIC PENDULUM-EXACT METHOD

**Objectives:**
**- Using the conservation of angular momentum and conservation of mechanical energy in a ballistic pendulum to find the launch velocity of a projectile.**
**- Using the conservation of linear momentum and conservation of mechanical energy in a ballistic pendulum to find the launch velocity of a projectile.**

***>>> Material Used >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>***

## *PROJECTILE LAUNCHER*

| Item | Code | Quant. | Unit | Description |
|---|---|---|---|---|
| 01 | 62002176 | 01 | Un. | CLAMP "C" |
| 02 | 62005751 | 01 | Un. | CANNON LAUNCHER HOLDER |
| 03 | 62002015 | 01 | Un. | CANNON (PROJECTILE LAUNCHER) |
| 04 | 62001074 | 01 | Un. | STEEL BALL Ø25MM |
| 06 | 62005177 | 01 | Un. | TUBE FOR CANNON COMPRESSION |
| 07 | 48005003 | 02 | Un. | BUTTERFLY NUT (CANNON'S FASTENER) |
| 08 | 50001004 | 02 | Un. | FLAT WASHER (CANNON'S FASTENER) |
| 09 | 62005317 | 01 | Un. | CANNON'S FASTENER |
| 13 | 03003011 | 01 | Un. | TAPE MEASURE 05M |
| 14 | 62005274 | 01 | Un. | PLUMB BOB WITH MAGNETIC FIXING |

## *Ballistic Pendulum Accessories(sold separately)*

| Item | Code | Quant. | Unit | Description |
|---|---|---|---|---|
| 01 | 62005611 | 02 | Un. | 50G CYLINDER MASS |
| 03 | 53003001 | 01 | Un. | PENDULUM'S TOWER THUMB SCREW'S NUT |
| 04 | 53001009 | 01 | Un. | PENDULUM'S THUMB SCREW |
| 04 | 53004002 | 01 | Un. | METAL HANDLE M3X10 |
| 05 | 62002055 | 01 | Un. | BALLISTIC PENDULUM |
| 06 | 62002182 | 01 | Un. | PENDULUM'S TOWER |

***>>> Experimental Procedures>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>***

Do not look directly at the cannon exit, as it may be charged!

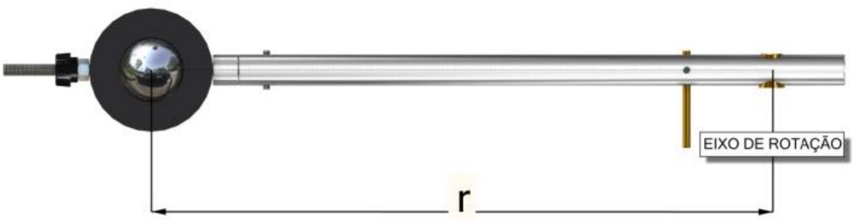
**1.** Assemble the launcher as in the previous experiment.

**2.** Measure the mass m of the metal ball and the mass M of the set (ballistic pendulum + ball).

*m = 0,064 kg   M = 0,294 kg*

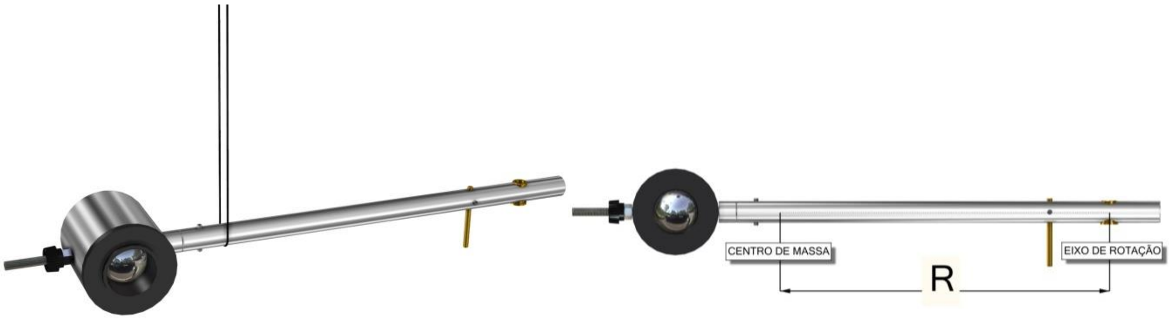
**3.** Measure the distance r from the center of the ball to the axis of rotation of the pendulum.

*r =0,255 m*



**4.** Insert the ball into the pendulum receiver and use a line to find the center of mass position of the set. Slide the line holding the pendulum until it remains in horizontal balance.

**5.** Measure the distance R from the position of the center of mass to the axis of rotation.




*R = 0,204 m*

**6.** Attach the pendulum to the launcher support and make the necessary adjustments.

**7.** For the correct positioning of the pendulum the following steps are suggested:

- Keep the pendulum free in vertical.
- Carefully approach the cannon until it softly engages the mouth of the pendulum trimmer.
- Tighten the cylinder locking screws.

With these procedures the ball, when launched, will always be correctly picked up by the pendulum receiver.

**8.** Place the steel ball in the projectile launcher and compress the spring to the second stage.

**9.** Adjust the pendulum position to align correctly into the launcher's mouth.

**10.** Adjusting the zero of the angle marker.

**11.** Trigger the cannon and note the angle value reached by the pendulum.

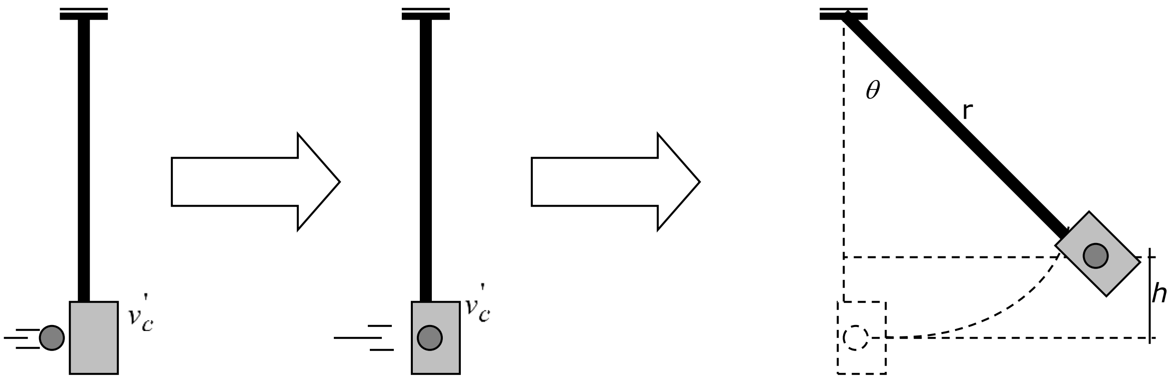
**12.** Fire some measurements of the reach angle of the pendulum.

**13.** Choose the most repeated measure and adjust the indicator to about four degrees less than that angle.

**14.** Repeat the launch three times and calculate the average value of the angle.

Table 1

| $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_{average}$ |
|---|---|---|---|
| 45,0º | 44,5º | 44,5º | 44,70º |




(a) (b) (c)

**15.** Assemble the pendulum without the launcher so it can swing freely.

**16.** Use a timer and measure the time of 20 complete oscillations of small amplitude. Repeat the procedure for at least three times and note the time value (t = 20T) for the 20 oscillations in the table.

**>>>Theoretical Foundations>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>**

The height $h$ reached by the center of mass of the pendulum is as shown in (c):

$$h = R(1 - cos\theta)$$

The potential energy stored in the system at the (c)

$$U = M \cdot g \cdot h = MgR(1 - cos\theta)$$

The kinetic energy immediately after the collision equals the potential energy at the point where the pendulum reaches the largest angle, and therefore:

$$U = K_d = M \cdot g \cdot h = MgR(1 - cos\theta)(1)$$

The kinetic energy $K_d$ and the angular momentum $L_d$ of the system immediately after the collision of the ball with the pendulum are given by the equations:

$K_d = \frac{I \cdot \omega^2}{2}$ and $L_d = I_{conj} \cdot \omega$

By combining the two equations we can obtain the relation between $L_d$ and $K_d$:

$$L_d = \sqrt{2 \cdot I_{conj} \cdot K_d} \qquad (2)$$

The angular momentum of the system immediately before the collision ($L_a$) is restricted to the angular momentum of the ball, since at that moment the pendulum is at rest:

$$L_a = I_{ball} \cdot \omega = m \cdot r^2 \omega = m \cdot r \cdot v_0 \quad (3)$$

Where: $\begin{cases} m - mass\ of\ the\ ball \\ r - distance\ from\ the\ ball\ to\ the\ axis\ of\ rotation \\ v_0 - velocity\ of\ the\ ball\ before\ the\ collision \end{cases}$

Considering that the angular momentum L of the system is conserved and by combining (2) and (3):

$$L_{before\ collision} = L_{after\ collision}$$

$$m \cdot r \cdot v_0 = \sqrt{2 \cdot I_{set} \cdot K_d} \qquad (4)$$

By combining (1) and (4), we obtain: $v_0 = \frac{1}{m \cdot r}\sqrt{2 \cdot I_{set} \cdot M \cdot g \cdot R(1 - cos\theta)}$ (5)

In the angular displacement of the pendulum the torque: $\tau = I_{set} \cdot \alpha = -R \cdot M \cdot g \cdot sen\theta$

For small angles this equation can be written: $\alpha = \frac{d^2\theta}{dt^2} = -\frac{MgR}{I_{set}} \cdot \theta$

Which is similar to a simple harmonic motion equation $\alpha = -\frac{k}{m}x = -\omega^2 x$

What allows us to write: $\omega^2 = \frac{MgR}{I} \rightarrow I_{conj} = \frac{MgR}{\omega^2} \rightarrow I_{conj} = \frac{MgRT^2}{4\pi^2}$(6)

The period T can be obtained by making the pendulum perform small oscillations.

### >>> *Analysis of Results and Conclusions* >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

**17.** Calculate the average value of the period T of the pendulum.

Table 2

| *Event* | *1* | *2* | *3* | *Average value* |
|---|---|---|---|---|
| *Time of 20 oscillations (s)* | *21,47* | *21,46* | *21,32* | *21,42* |
| *Period T (s)* | *1,074* | *1,073* | *1,066* | *1,071* |

**18.** Use the equation (6) and calculate the moment of inertia of the set $I_{set}$.

$$I_{set} = \frac{0,294 \times 9,78 \times 0,204 \times 1,071^2}{4\pi^2} \rightarrow I_{set} = 1,704 \cdot 10^{-2}\ kg \cdot m^2$$

**19.** Use the equation (5) and calculate the ball velocity immediately before launch.

$$v_0 = \frac{1}{0,064 \cdot 0,255}\sqrt{2 \times 1,704 \cdot 10^{-2} \times 0,294 \times 9,78 \times 0,204(1 - cos44,7^o)}$$

$$v_0 = 4,65\ m/s$$

**20.** Calculate the percentage difference between the two results found: $d\% = \frac{|A-B|}{\frac{A+B}{2}} \times 100\%$

$$d\% = \frac{|4,65 - 4,46|}{\frac{4,65 + 4,46}{2}} \times 100\% = 4,2\%$$

**21.** Does the percentage difference obtained confirm the validity of the conservation principle of angular momentum? Justify.

Considering that the percentage difference is much lower than the tolerance of up to 10%, it can be affirmed that the experiment ratifies the validity of the conservation of angular momentum. It is observed in this experiment that the rotational inertia considerations allowed to reach incomparably better results than in the approximate method experiment. Therefore, this procedure is called the "exact method".

**22.** What can be concluded about the energy conservation in the collision?

There are two moments when you consider the energy of the system:

**a.** The kinetic energy of the ball immediately before the collision with the pendulum:

$$K_{antes} = \frac{m_{esf} \cdot v_o^2}{2} = \frac{0,064 \times 4,46^2}{2} = 0,637\,J$$

**b.** The kinetic energy of the set immediately after the collision:

$$K_{depois} = \frac{M \cdot v_{conj}^2}{2} = \frac{0,294 \times 1,07^2}{2} = 0,168\,J$$

$$\Delta K = 0,168 - 0,637 = -0,469\,J$$

$$\Delta K = 0,168 - 0,637 = -0,469\,J$$

There was a loss of 74% of energy which shows that there was no energy conservation and therefore, it is an inelastic collision. This was already expected, as the bodies remained united after the collision.

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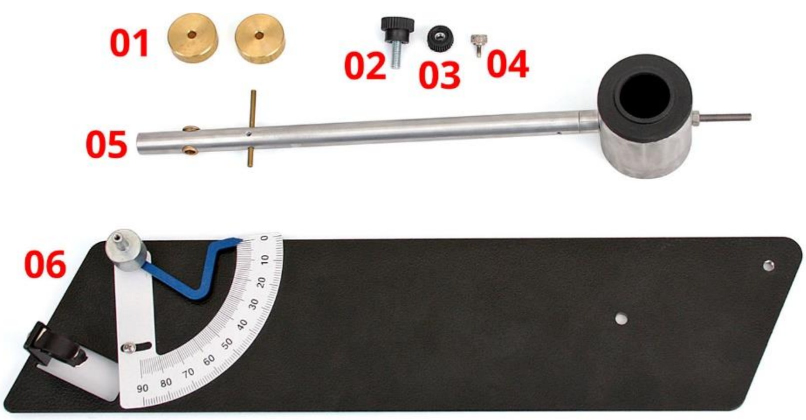
# COMPOSITION


## *Projectile Launcher*

| Item | Code | Quant. | Unit | Description |
|---|---|---|---|---|
| 01 | 62002176 | 01 | Un. | CLAMP "C" |
| 02 | 62005751 | 01 | Un. | CANNON HOLDER |
| 03 | 62002015 | 01 | Un. | CANNON (PROJECTILE LAUNCHER) |
| 04 | 62001074 | 02 | Un. | STEEL BALL Ø25MM |
| 05 | 62005120 | 01 | Un. | PLASTIC BALL Ø25MM |
| 06 | 62005177 | 01 | Un. | TUBE FOR CANNON COMPRESSION |
| 07 | 48005003 | 02 | Un. | BUTTERFLY NUT (CANNON'S FASTENER) |
| 08 | 50001004 | 02 | Un. | FLAT WASHER (CANNON'S FASTENER) |
| 09 | 62005317 | 01 | Un. | CANNON'S FASTENER |
| 10 | 62001023 | 01 | Un. | SENSORS/SPHERE SHOCK'S HOLDER |
| 11 | 62002056 | 01 | Un. | BALL'S MAGNETIC FIXING (FOR COLLISION) |
| 12 | 53003001 | 01 | Un. | FASTENER FOR BALL'S MAGNETIC FIXING |
| 13 | 03003011 | 01 | Un. | TAPE MEASURE 05M |
| 14 | 62005274 | 01 | Un. | PLUMB BOB WITH MAGNETIC FIXING |




## *Ballistic Pendulum Accessories (sold separately)*

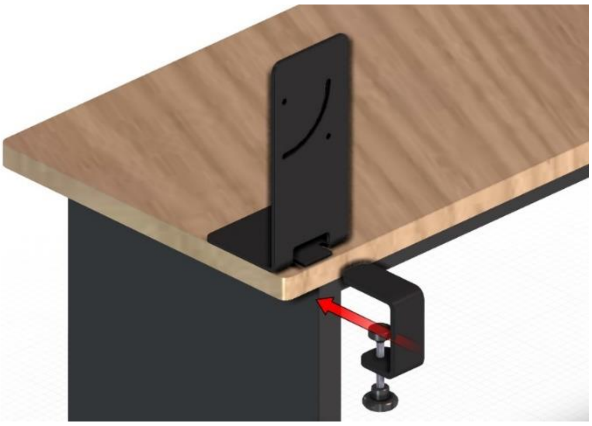
| Item | Code | Quant. | Unit | Description |
|---|---|---|---|---|
| 01 | 62005611 | 02 | Un. | 50G CYLINDER MASS |
| 02 | | 01 | Un | PENDULUM'S TOWER THUMB SCREW |
| 03 | 53003001 | 01 | Un. | PENDULUM'S TOWER THUMB SCREW'S NUT |
| 04 | 53001009 | 01 | Un. | PENDULUM'S THUMB SCREW |
| 05 | 62002055 | 01 | Un. | BALLISTIC PENDULUM |
| 06 | 62002182 | 01 | Un. | PENDULUM'S TOWER |

# ACCESSORIES (SOLD SEPARATELY)

| Code | Quant. | Unit | Description | Picture |
|---|---|---|---|---|
| 62001226 | 01 | Un. | DIGITAL TIMER AZB-30 USB |  |
| 62001255 | 01 | Un. | PHOTOGATE TIMER LITE |  |
| 62001201 | 02 | Un. | PHOTOELECTRIC SENSOR PGS-D10 |  |
| 04002037 | 01 | Un. | FLIGHT TIME SENSOR TFS-D10 |  |
| 30002014 | 06 | Un. | PAIR OF DOCKABLE RODS Ø12,7MMX400MM |  |
| 29003002 | 04 | Un. | BIG TRIPOD |  |
| 31003001 | 04 | Un. | METALLIC CLIP AZB-027 |  |
| 02005006 | 04 | Un. | LAB CLAMP WITH RING |  |

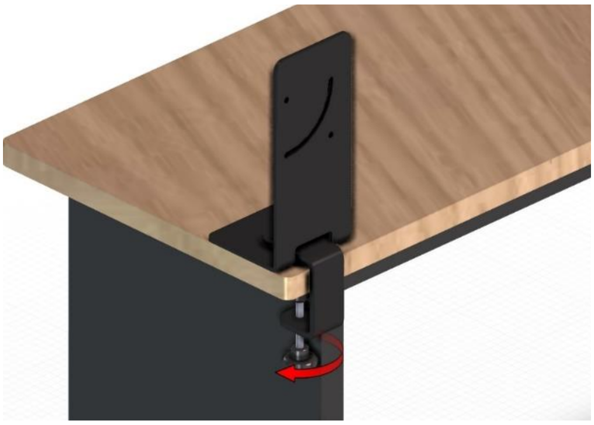
# ASSEMBLY SCHEMES

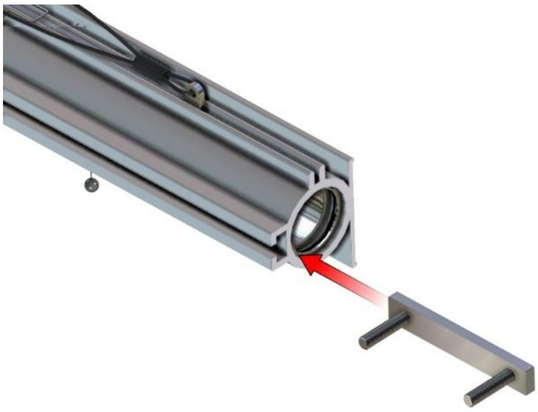
## PROJECTILE LAUNCHER



Position the CANNON HOLDER on the corner of a table. Then insert the clamp "C" as shown in the figure.

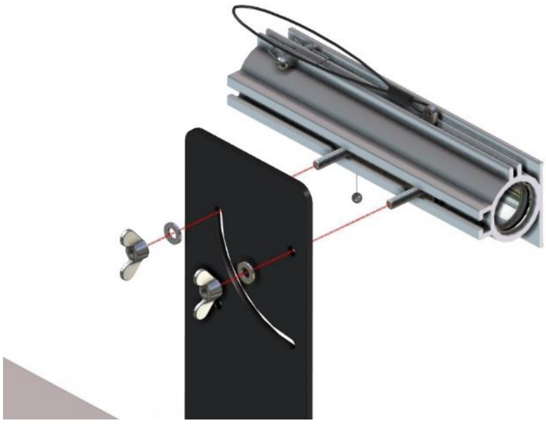
Tighten the clamp knob until the CANNON HOLDER is steady.

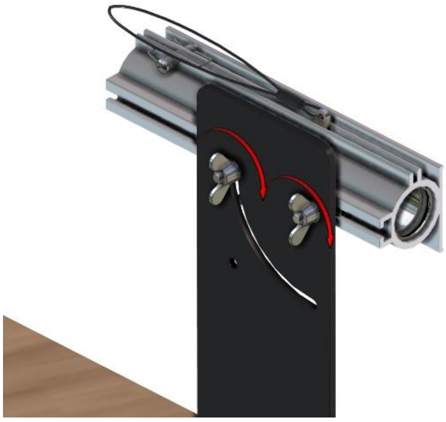




Position the CANNON'S FASTENER inside the cannon rail.

Place the cannon in the support "L" as shown in the figure and fasten it with FLAT WASHER and BUTTERFLY NUT (CANNON’S FASTENER) without tightening to the end.

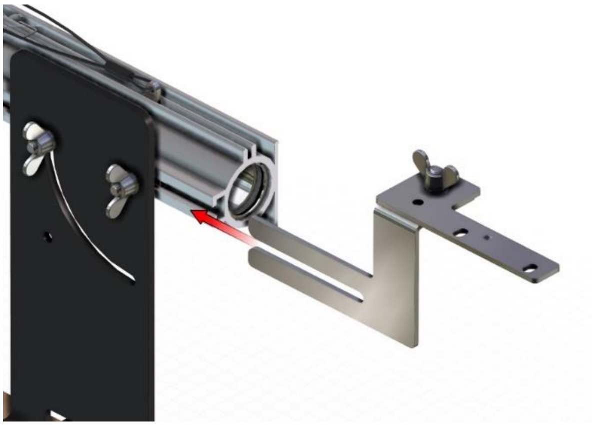




Position the cannon at the desired angle and then tighten the BUTTERFLY NUT.

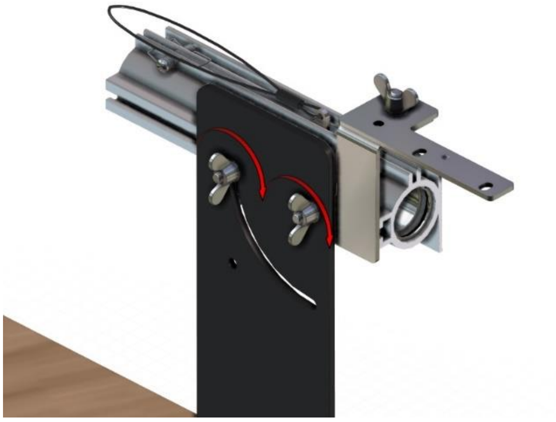
Ready, the cannon is assembled and ready to be used.

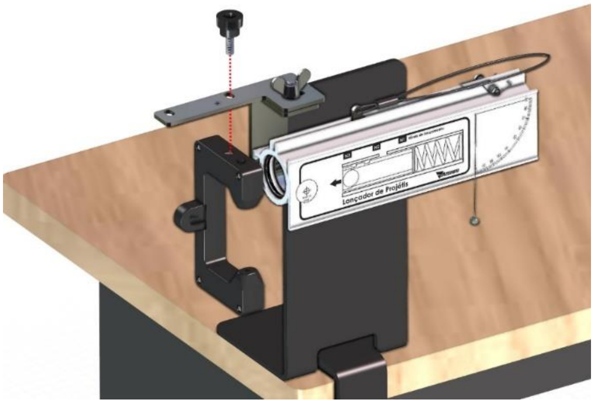
# PHOTOELECTRIC SENSORS PGS-D10



Loosen the BUTTERFLY NUT slightly and insert the SENSORS/SPHERE SHOCK'S HOLDER as shown in figure.

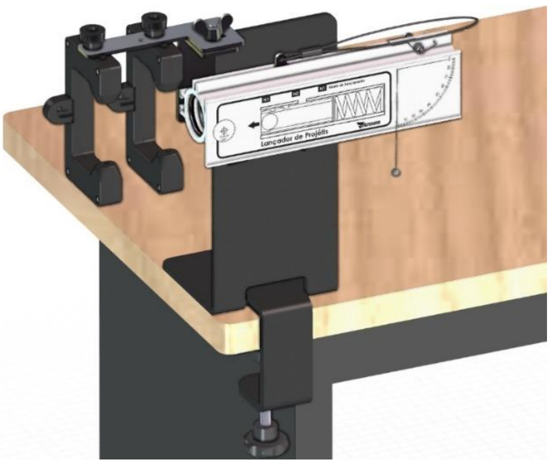
With the support in place, tighten the BUTTERFLY NUT again.

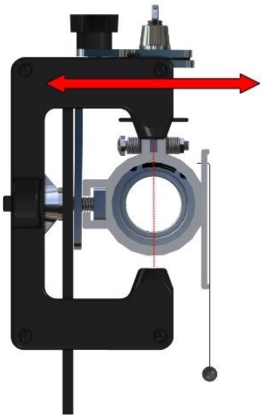




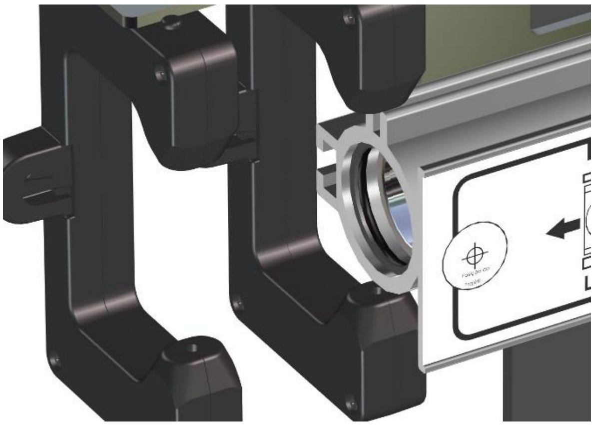
Fix the sensors as shown in the figure.

The image on the right shows the two sensors assembled.



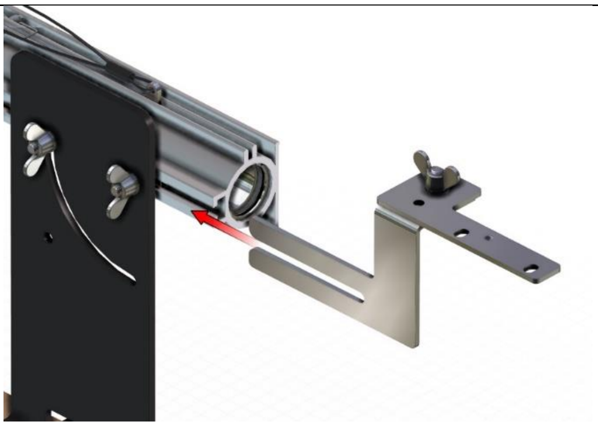


The figure shows the central alignment of the sensor.



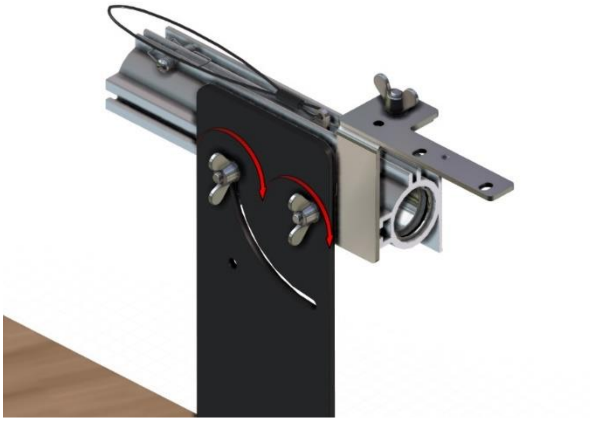
The image shows the positioning of the timer "start" sensor. It should be well positioned at the exit of the cannon

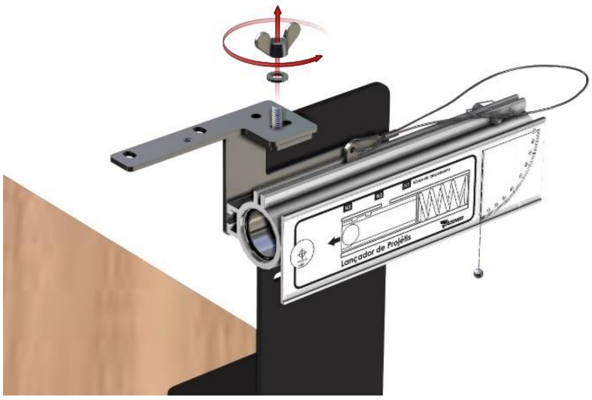
# BALL'S MAGNETIC FIXING



Loosen the BUTTERFLY NUT slightly and insert the SENSORS/SPHERE SHOCK'S HOLDER as shown in figure.

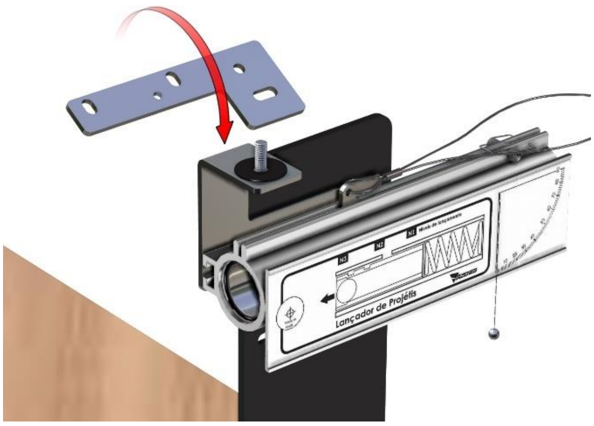
Then tighten the BUTTERFLY NUT again.

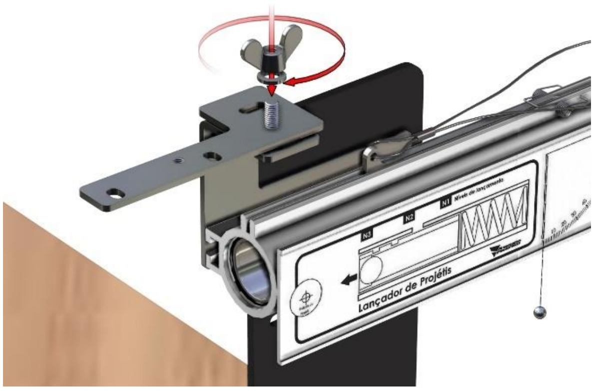




Remove the BUTTERFLY NUT to release the "L" shaped part..

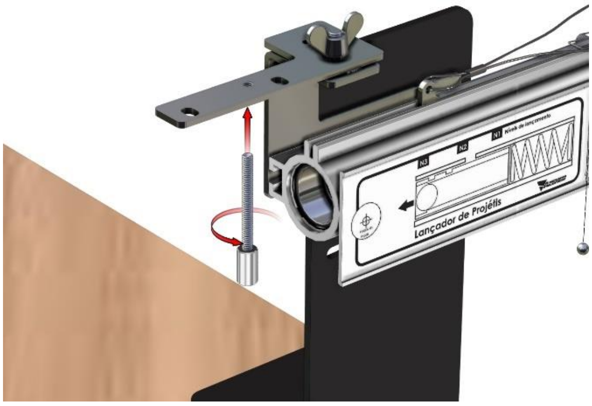
Reverse its position.

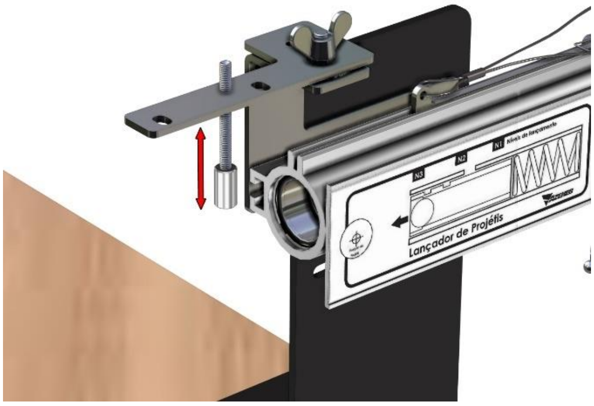




Replace the BUTTERFLY NUT.

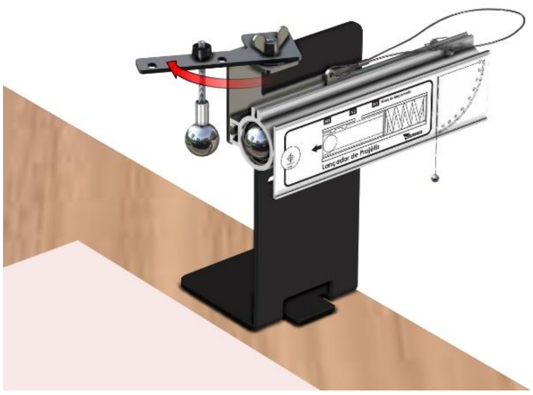
Thread the BALL’S MAGNETIC FIXING.



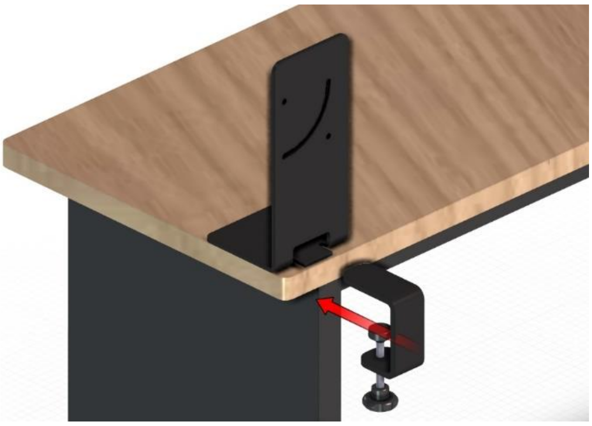


Adjust the height of BALL’S MAGNETIC FIXING. If necessary place one ball in the cannon and another in the fixing to check the alignment. The two balls must be aligned by a center line.

To adjust the collision angle, loosen the butterfly nut slightly and then move the support "L" to the desired position and retighten the butterfly nut.

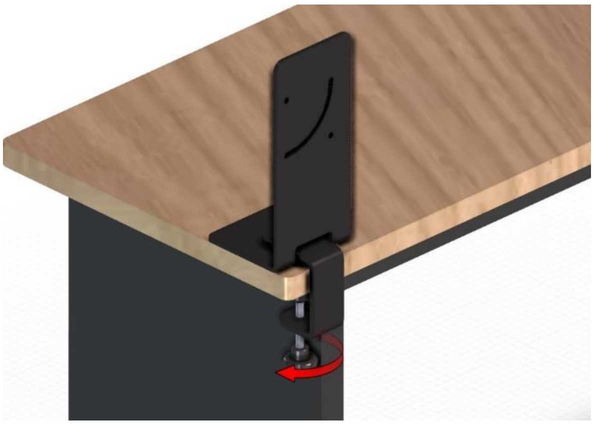


# BALLISTIC PENDULUM



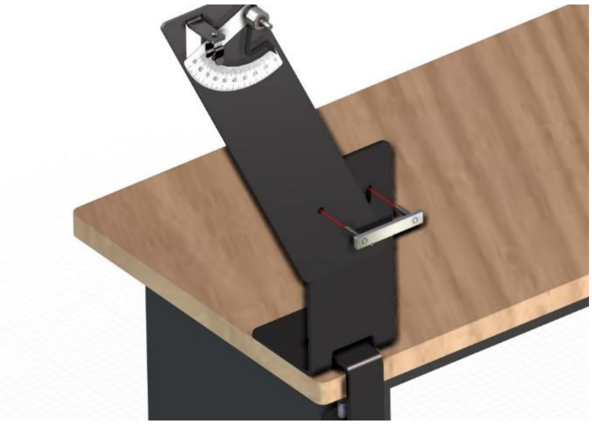
Position the CANNON HOLDER in one of the corners of the table and fit the "C" clamp. If the table surface is slick, use a thin rubber cloth under the HOLDER.

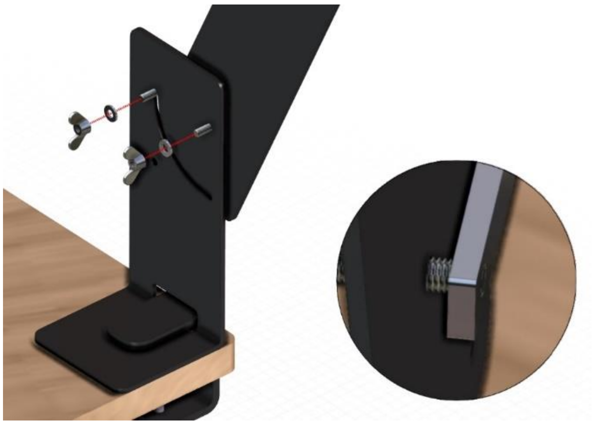
Tighten "C" clamp .



Position the PENDULUM’S TOWER.

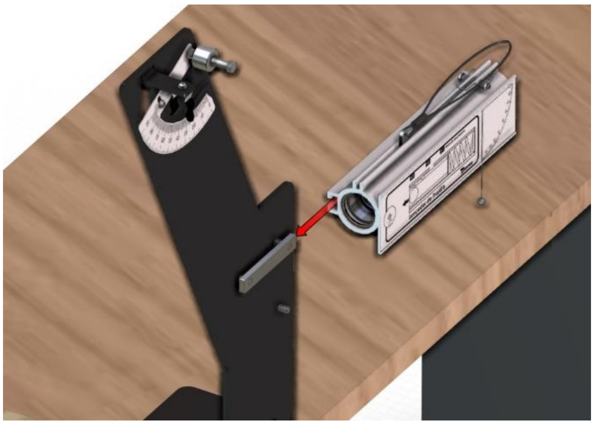
Insert the CANNON’S FASTENER into the holes indicated in the figure.





Fasten the washers and butterfly nuts. Do not tighten until the end, it is necessary to leave a gap to fit the cannon.

Lock the PENDULUM'S TOWER by fastening the PENDULUM'S TOWER THUMB SCREW as shown in the figure.

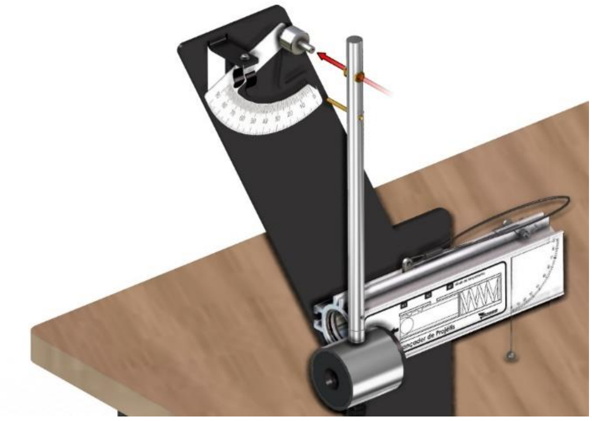


Slide the cannon as shown in the figure.

Tighten the butterfly nuts to fix the cannon.

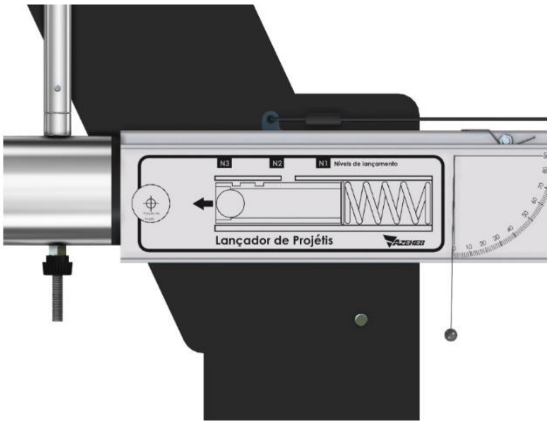
Remove the PENDULUM’S THUMB SCREW.

Place the BALLISTIC PENDULUM.



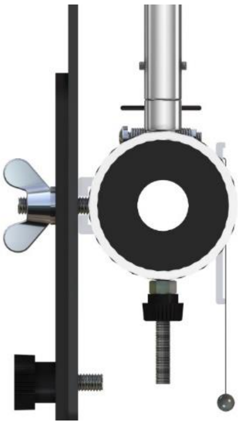
Reattach the PENDULUM’S THUMB SCREW.

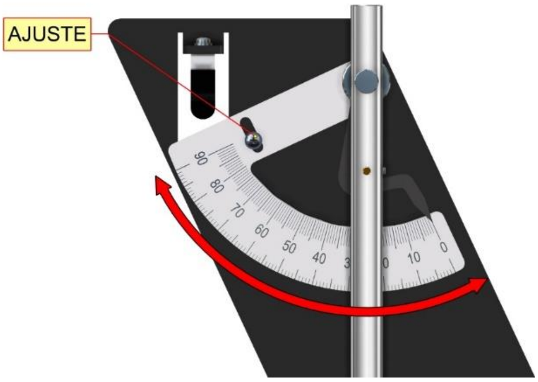
Position the cannon launcher.




The positioning must be as seen in the figure.

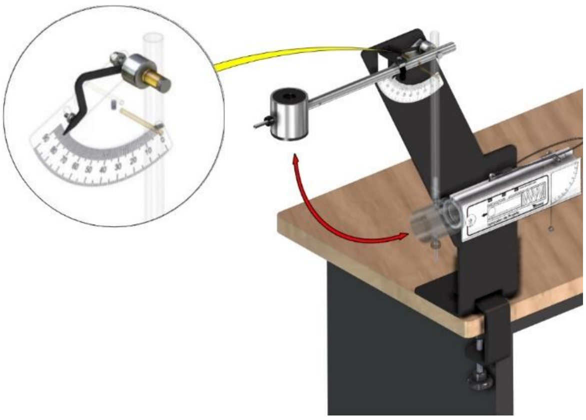
Observe ballistic pendulum alignment.

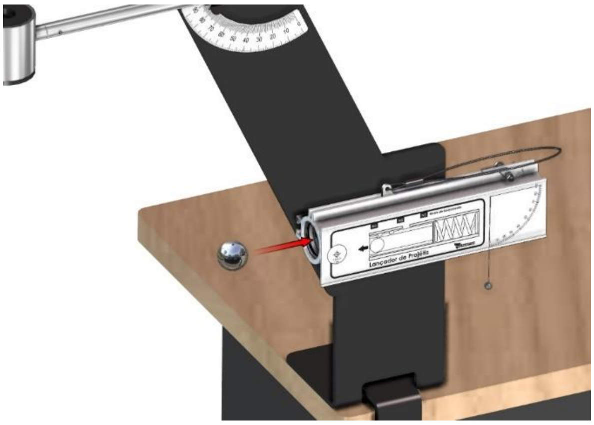





Turn the angle marker pointer until it touches the ballistic pendulum arm. If angle is not 0º, loosen the screw and adjust the scale to 0º.

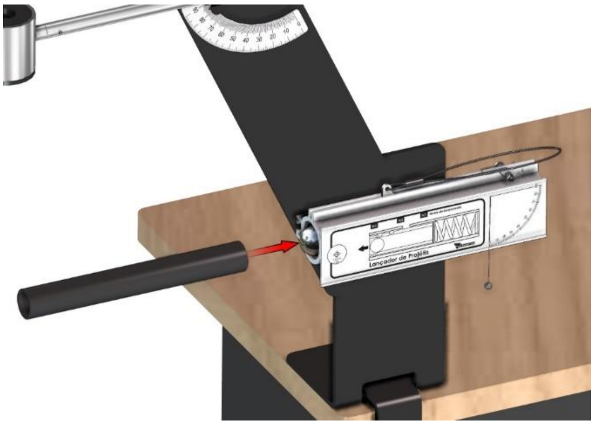
With the ballistic pendulum assembled and adjusted, raise the pendulum until the arm attaches to the cramp at the top.





Place the ball on the cannon.

Using the plastic tube, push the ball to charge the cannon.



With the cannon charged, lower the ballistic pendulum until it snaps into the cannon's mouth and position the angle indicating pointer at 0º.
Done, now just shoot the cannon.

# EXPERIMENTS

## EXPERIMENT 01 – HORIZONTAL PROJECTILE LAUNCH

Objectives:
- Recognizing the physical quantities involved in a horizontal projectile launch
- Checking the relationship between the physical quantities present in a horizontal launch.

>>> ***Material Used*** >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

| Item | Code | Quant. | Unit | Description |
|---|---|---|---|---|
| 01 | 62002176 | 01 | Un. | CLAMP "C" |
| 02 | 62005751 | 01 | Un. | CANNON LAUNCHER HOLDER |
| 03 | 62002015 | 01 | Un. | CANNON (PROJECTILE LAUNCHER) |
| 05 | 62005120 | 01 | Un. | PLASTIC BALL Ø25MM |
| 06 | 62005177 | 01 | Un. | TUBE FOR CANNON COMPRESSION |
| 07 | 48005003 | 02 | Un. | BUTTERFLY NUT (CANNON'S FASTENER) |
| 08 | 50001004 | 02 | Un. | FLAT WASHER (CANNON'S FASTENER) |
| 09 | 62005317 | 01 | Un. | CANNON'S FASTENER |
| 10 | 62001023 | 01 | Un. | SENSORS/SPHERE SHOCK'S HOLDER |
| 13 | 03003011 | 01 | Un. | TAPE MEASURE 05M |
| 14 | 62005274 | 01 | Un. | PLUMB BOB WITH MAGNETIC FIXING |
| XX | 62001226 | 01 | Un. | DIGITAL TIMER AZB-30 USB (*) |
| XX | 62001201 | 02 | Un. | PHOTOELECTRIC SENSOR PGS-D10 (*) |
| XX | 04002037 | 01 | Un. | FLIGHT TIME SENSOR TFS-D10 |

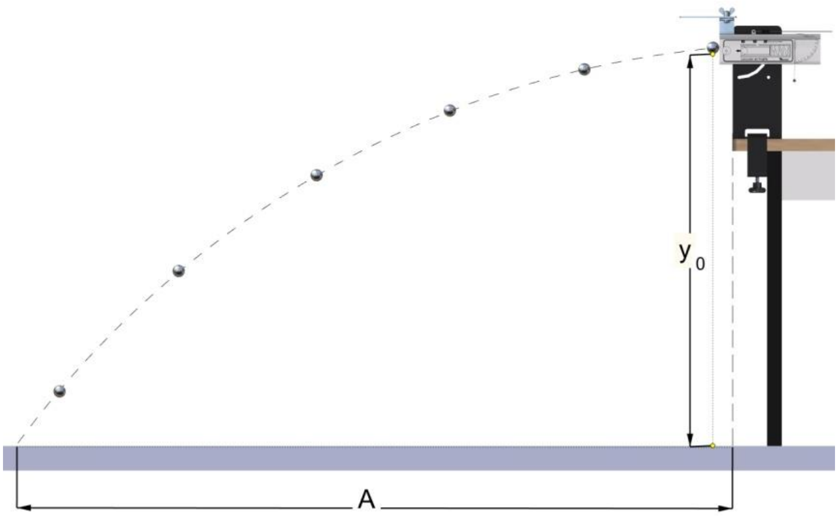
**(*) It does not accompany the product. It is sold separately.**

## Part I - Determining the horizontal launch velocity ($v_0$) using the reach measure (A) and the launch height ($y_0$)

>>> ***Experimental Procedures***>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

Do not look directly at the cannon exit, as it may be charged!

**1.** Assemble the launcher so that the projectile has space to move and fall on the floor, as shown in the figure.



2. Measure the launch height ($y_o$) in relation to the floor. The measurement shall be made from the lower part of the ball indicated at the exit of the cannon, to the floor, according to the figure.

3. With a plumb bob, mark on a paper sheet (scotch taped on the floor) the position (origin of horizontal displacement). The plumb bob must match the vertical passing through the center of the ball.

4. Place the plastic ball in the cannon and compress the spring to the second stage. Pull the trigger and observe where the ball touches the floor.

5. At the dropping point of the ball place a paper sheet (scotch tape it as well) with a carbon paper over it.

6. Repeat the launch 10 times and measure the reached distance range A using the measure tape. Complete the table 1 with the values.

Table 1

| N | Reached distance (m) |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| Average value | |

**>>> *Analysis of Results and Conclusions* >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>**

7. What are the active forces in the sphere after the launch?

**8.** Classify the projectile motion according to the two directions (vertical and horizontal).

**9.** Which equation must be used in each motion?

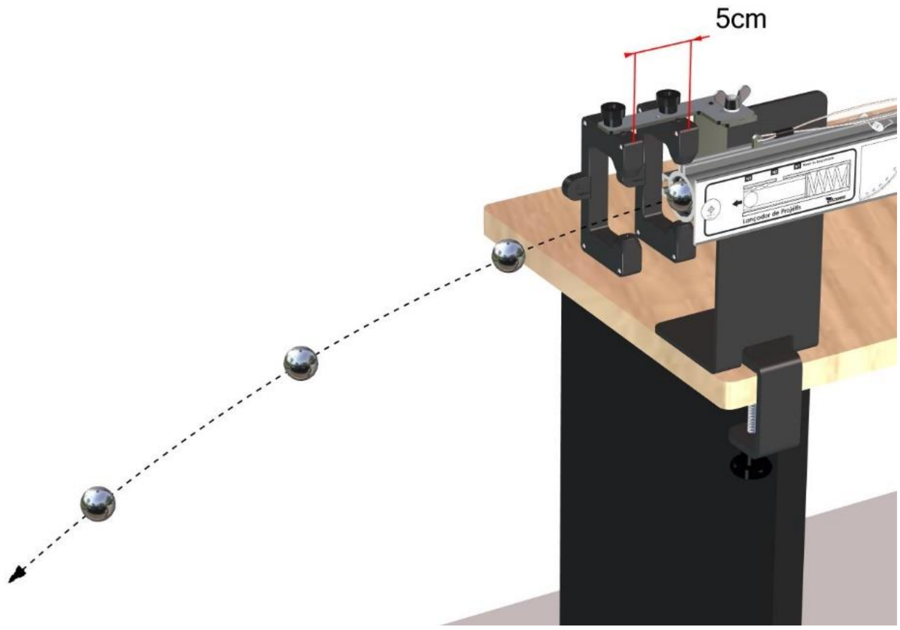
**10.** Use the horizontal launch equations and calculate the time in the air.

# Part II - Determination of horizontal velocity of launch ($v_o$) using the measurement of projectile passage time by two sensors

***>>> Experimental Procedures>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>***

Do not look directly at the cannon exit, as it may be charged!

**1.** Assemble the projectile launcher as shown in the figure (sensors are positioned approximately 5cm apart):



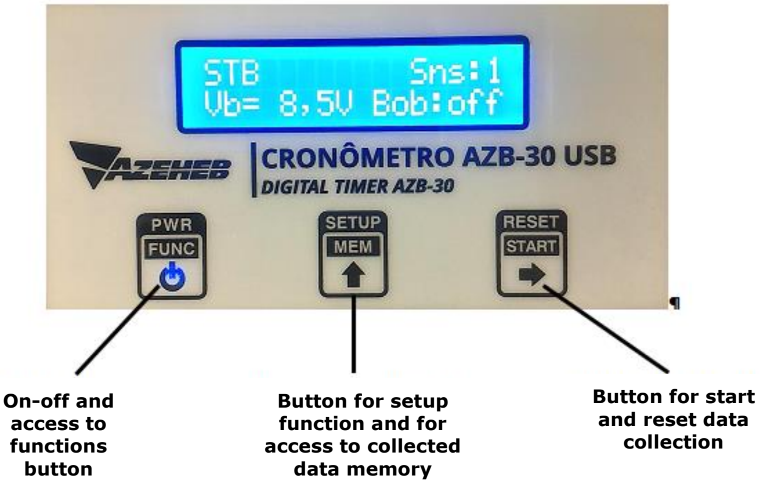

**2.** Align correctly the sensors.

**3.** Connect the sensors to the timer.

**4.** Measure the distance between the two sensors centers.

**5.** Turn on the timer and check if it has identified the sensors. Test the sensors by interrupting the infrared passage. An intermittent exclamation mark (!) should appear in the upper right corner of the screen indicating that the sensors are identified.

- When you turn on the timer, it displays the STANDBY screen and should show:
- STB
- Vb: 8,5V – Output Voltage.
- Sns :2– Number of sensors identified.
- Bobbin: off - indicating the coil is off.




**6.** Select function 1 by clicking FUNC. The display will show the function 1 F1. The display should read "[F1]" "Btn nS:2". In this function the timer will be using two sensors, and the chronometer will present two time measurements. (See information in the timer manual).


**7.** Schedule the timer for data acquisition by starting the time counting on the first sensor and interrupting the second one.

**8.** Press SETUP, press for a few seconds until the display shows CFG, use the key (↓) and adjust from Btn to Sns. Click START to end programming. On the nS: 1 screen, it indicates that the timer will only take one measure of time.


**9.** As the objective is only to measure the time interval between the two sensors, collect the ball just after passing through the second sensor, using a cardboard box.

**10.** Position the plastic ball in the second stage of the launcher.

**11.** By pressing START on the display it will show the "standby" signal (*) flashing. By pressing START again, the signal (*) is in the operating mode, waiting for the ball to pass through the sensor.

**12.** After the launch the screen shows the time interval value of the ball passing through the two sensors.

**13.** Write down the time measured in the table 1. Repeat data collection at least 5 times.

**14.** By pressing START the time measurement of the screen is sent to memory. For a new measure go back to item 11. Complete the table 1.

Table 1

| N | Δx (m) | Δt (s) | *v (m/s)* |
|---|---|---|---|
| 1 | 0,05 | | |
| 2 | 0,05 | | |
| 3 | 0,05 | | |
| 4 | 0,05 | | |
| 5 | 0,05 | | |
| | Average value | | |

>>> ***Analysis of Results and Conclusions*** >>>>>>>>>>>>>>>>>>>>>>>>>>>

**15.** Use the equation below and calculate the ball velocity through the sensors:

$$v = \frac{\Delta x}{\Delta t}$$

**16.** Determine the average value of the launch velocity.

**17.** Compare the experimental values of the launch velocities found by processes (a) and (b). Was there any significant difference between the values found?

| *Method* | *By the reach measure and launch height ($v_{o1}$)* | *By the time of passage between two sensors ($v_{o2}$)* |
|---|---|---|
| *Velocity (m/s)* | | |

**18.** Calculate the average launch velocity of the two procedures.

**19.** Calculate the percentage error: $\boldsymbol{e}\% = \frac{|\boldsymbol{v_{01}} - \boldsymbol{v_{02}}|}{\frac{\boldsymbol{v_{01}} + \boldsymbol{v_{02}}}{2}}$

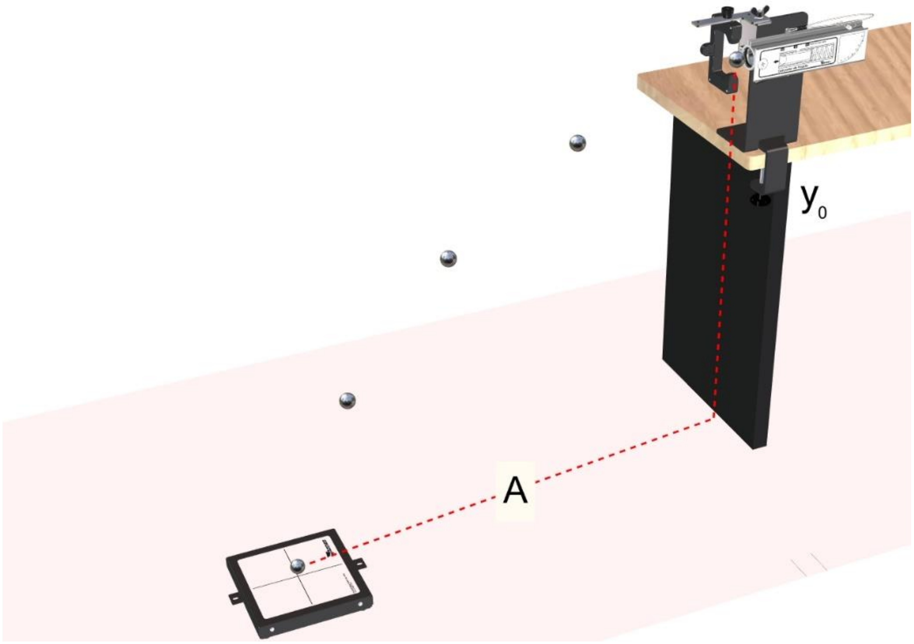
**20.** Justify possible inconsistencies between results.

# Part III - Determination of horizontal velocity of launch ($v_o$) using the measure of the reach (A) and the time of the projectile in the air

**>>> *Experimental Procedures*>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>**

Do not look directly at the cannon exit, as it may be charged!

1. Assemble the cannon as shown in the figure (with a sensor and the flight time sensor). Cover the flight sensor with a carbon paper to mark the point where the ball touches the lowest point of the path.




2. Connect the sensor and platform to the timer.

3. Adjust the sensor position as close as possible to the launcher's mouth.

4. Turn on the timer and check if it has identified the sensors and the platform. Test the sensors by interrupting the infrared passage. An intermittent exclamation mark (!) Should appear in the upper right corner of the screen indicating that the sensor 1 are identified. Tap the sensor briefly, the display shows signal 2 (!) indicating that the timer has identified the flight sensor.

   – When you turn on the timer, it displays the STANDBY screen and should show:
   – STB
   – Vb: 8,5V – Output Voltage.
   – Sns :2– Number of sensors identified.
   – Bobbin: off - indicating the coil is off.

5. Select function 1 by clicking FUNC. The display will show the function 1 F1. The display should read "[F1]" "Btn nS:2". In this function the timer will be using two sensors, and the timer will present on time measurement. (See information in the timer manual).


**6.** Schedule the timer for data acquisition by starting the time counting on the first sensor and interrupting the second one.

**7.** Press SETUP, press for a few seconds until the display shows CFG, use the key (↓) and adjust from Btn to Sns. Click START to end programming. On the nS: 1 screen, it indicates that the timer will only take one measure of time.


**8.** Position the plastic ball in the second stage of the cannon.

**9.** By pressing START on the display it will show the "standby" signal (*) flashing. By pressing START again, the signal (*) is in the operating mode, waiting for the ball to pass through the sensor.

**10.** After the ball touches the flight sensor, the time counting stops.

**11.** Write down in the table the measured time. Repeat the procedure at least 5 times and calculate the average reach distance (A). To measure the reach, observe the following precautions:

- Place a plumb bob from the center of the sensor to the floor. Mark this reference.
- The reach shall be measured from the reference point on the floor to the point where the ball touches the platform.

**12.** By pressing START the time measurement of the screen is sent to memory. For a new measure go back to item 11. Complete the table 1.

Table 1

| N | A (m) | Δt (s) | v (m/s) |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| | Average value | | |

### >>> ***Analysis of Results and Conclusions*** >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

**13.** Use the follow equation and calculate the ball velocity through the sensors:

$$v = \frac{A}{\Delta t}$$

**14.** Determine the average value of the launch velocity.

**15.** Compare the experimental values of the launch velocities found by processes (a) and (b). Was there any significant difference between the values found?

| *Method* | *By the reach measure and launch height ($v_{o1}$)* | *By the time of passage between two sensors ($v_{o2}$)* | *By the residence time in the air ($v_{o3}$)* |
|---|---|---|---|
| *Velocity (m/s)* | | | |

**16.** Calculate the average value of the launch velocity found by the two procedures.

# EXPERIMENT 02 – PROJECTILE OBLIQUE LAUNCH

**Objectives:**
**- Recognizing the physical quantities involved in a projectile oblique launch.**
**- Check the relationship among the physical quantities present in an oblique launch.**

>>> ***Material Used*** >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

| Item | Code | Quant. | Unit | Description |
|---|---|---|---|---|
| 01 | 62002176 | 01 | Un. | CLAMP "C" |
| 02 | 62005751 | 01 | Un. | CANNON LAUNCHER HOLDER |
| 03 | 62002015 | 01 | Un. | CANNON (PROJECTILE LAUNCHER) |
| 05 | 62005120 | 01 | Un. | PLASTIC BALL Ø25MM |
| 06 | 62005177 | 01 | Un. | TUBE FOR CANNON COMPRESSION |
| 07 | 48005003 | 02 | Un. | BUTTERFLY NUT (CANNON'S FASTENER) |
| 08 | 50001004 | 02 | Un. | FLAT WASHER (CANNON'S FASTENER) |
| 09 | 62005317 | 01 | Un. | CANNON'S FASTENER |
| 10 | 62001023 | 01 | Un. | SENSORS/SPHERE SHOCK'S HOLDER |
| 13 | 03003011 | 01 | Un. | TAPE MEASURE 05M |
| 14 | 62005274 | 01 | Un. | PLUMB BOB WITH MAGNETIC FIXING |
| XX | 62001226 | 01 | Un. | DIGITAL TIMER AZB-30 USB (*) |
| XX | 62001201 | 02 | Un. | PHOTOELECTRIC SENSOR PGS-D10 (*) |
| XX | 04002037 | 01 | Un. | FLIGHT TIME SENSOR TFS-D10 |

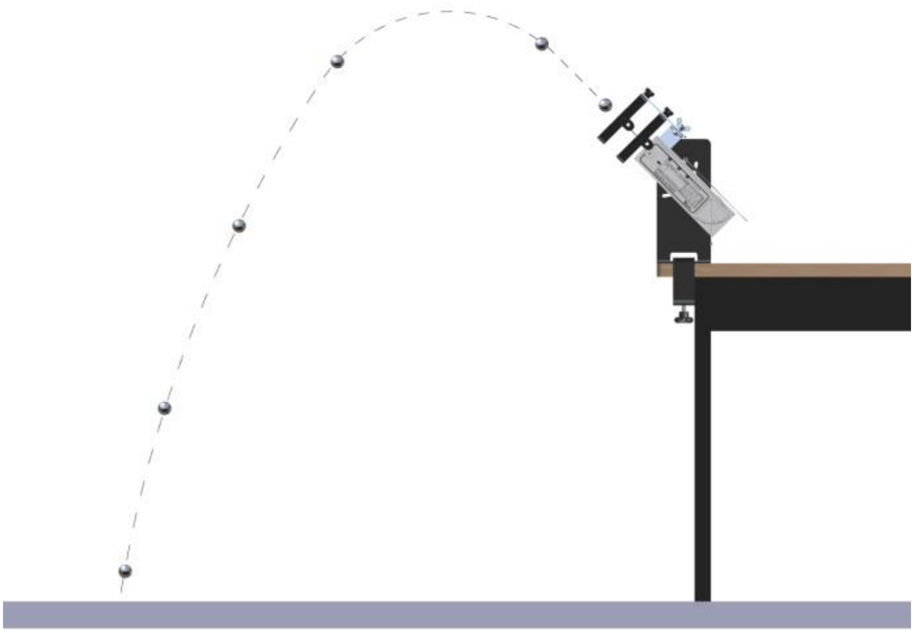
**(*) It does not accompany the product. It is sold separately.**

## Part I - Measure of Reach (A) and determining of launch velocity ($v_0$) using the measurement of the passage time between two sensors.

>>> ***Experimental Procedures***>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

Do not look directly at the cannon exit, as it may be charged!

**1.** Assemble the equipment as shown (the space between sensors is 5 cm):



2. Align sensors correctly and position the first of them as close as possible to the launcher's exit.

3. Connect the sensors to the timer.

4. Measure the distance between the two sensors centers.

5. Adjust the cannon launch position to an angle of 30º.

6. Measure the launch height ($y_0$) (from the bottom of the ball to the floor).

7. Launch the ball once to put a paper sheet and a carbon paper over it.

8. Select function 1 by clicking FUNC. The display will show the function 1 F1. The display should read "[F1]" "Btn nS:2". In this function the timer will be using two sensors, and the timer will present two time measurements. (See information in the timer manual).


9. Schedule the timer for data acquisition by starting the time counting on the first sensor and interrupting the second one.

10. Press SETUP, press for a few seconds until the display shows CFG, use the key (↓) and adjust from Btn to Sns. Click START to end programming. On the nS: 1 screen, it indicates that the timer will only take one measure of time.


11. Position the plastic ball in the second stage of the launcher.

12. By pressing START on the display it will show the "standby" signal (*) flashing. By pressing START again, the signal (*) is in the operating mode, waiting for the ball to pass through the sensor.

**13.** Fire the launcher. After the launch the screen shows the time interval value of the ball passing through the two sensors.

**14.** Measure the horizontal range of the ball (A) and the time interval value and note this data in the table. Repeat data collection at least 5 times.

**15.** By pressing START the time measurement of the screen is sent to memory. For a new measure go back to item 12. Complete the table 1.

Table 1

| N | A(m) | $t_p$ (s) | $v_o$ (m/s) |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| Average value | | | |

>>> ***Analysis of Results and Conclusions*** >>>>>>>>>>>>>>>>>>>>>>>>>>>>

**16.** Use the values in the table and calculate the average value of the measured reach - $A_{measured}$

$$A_{med} = \frac{\sum A_i}{N}$$

**17.** Use the equation below and table values to calculate the ball velocity through the sensors:

$$v = \frac{\Delta x}{\Delta t}$$

**18.** Determine the average value of the launch velocity ($v_o$).

**19.** Use the values obtained for the initial velocity ($v_o$), height ($y_o$) and launch angle ($\theta$) and determine the predicted reach value using the equation:

$$A = v_0 \cdot cos\theta \cdot \frac{v_0 \cdot sen\theta + \sqrt{v_0^2 \cdot sen^2\theta + 2gy_0}}{g}$$

**20.** Compare the predicted reach value with the measured value.

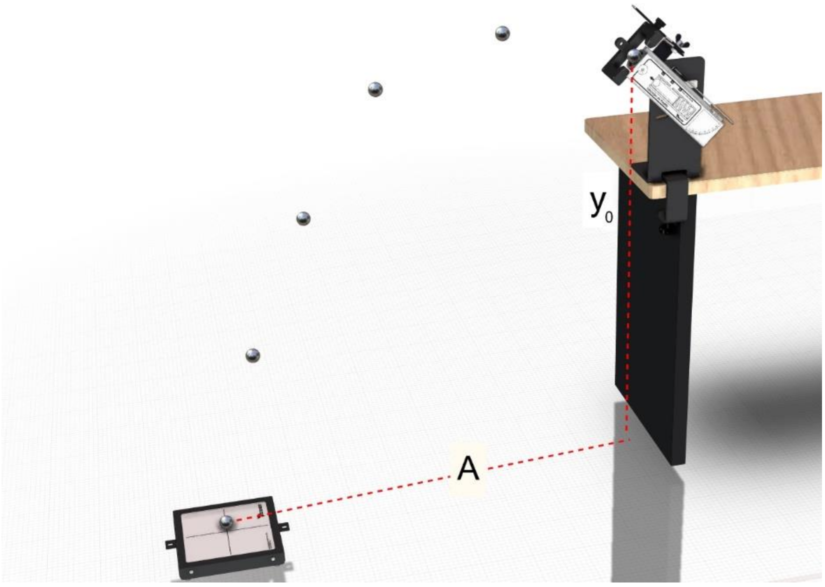
$$e\% = \frac{\left|A_{measured} - A_{predicted}\right|}{\frac{A_{measured} + A_{predicted}}{2}}$$

# Part II – Determining launch velocity ($v_o$) using the measure of reach (A) and the projectile residence time interval in the air.

>>> ***Experimental Procedures***>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

Do not look directly at the cannon exit, as it may be charged!

**1.** Assembly as shown in the figure (with a sensor and the flight time sensor).




**2.** Connect the sensors and the platform to the timer.

**3.** Adjust the sensor position as close as possible to the launcher's mouth.

**4.** Move the assembly to a launch at an angle of 60º.

**5.** Turn on the timer and check if it has identified the sensors and the platform. Test the sensors by interrupting the infrared passage. An intermittent exclamation mark (!) should appear in the upper right corner of the screen indicating that the sensor 1 are identified. Tap the sensor briefly; the display shows signal 2 (!) indicating that the timer has identified the flight sensor.

- When you turn on the timer, it displays the STANDBY screen and should show:
- STB
- Vb: 8,5V – Output Voltage.
- Sns :2– Number of sensors identified.
- Bobbin: off - indicating the coil is off.

**6.** Select function 1 by clicking FUNC. The display will show the function 1 F1. The display should read "[F1]" "Btn nS:2". In this function the timer will be using two sensors, and the chronometer will present two time measurements. (See information in the timer manual).


**7.** Schedule the timer for data acquisition by starting the time counting on the first sensor and interrupting the second one.

**8.** Press SETUP, press for a few seconds until the display shows CFG, use the key (↓) and adjust from Btn to Sns. Click START to end programming. On the nS: 1 screen, it indicates that the timer will only take one measure of time.


**9.** Position the plastic ball in the second stage of the launcher and perform some launches to identify the probable position of the platform.

**10.** By pressing START on the display it will show the "standby" signal (*) flashing. By pressing START again, the signal (*) is in the operating mode, waiting for the ball to pass through the sensor.

**11.** After the ball touches the flight sensor, the time counting stops.

**12.** Note the time interval value provided by the timer in the table.

**13.** Measure the reach (A) and note it in the table. To measure the reach, observe the following precautions:

- Place a plumb bob from the center of the sensor to the floor. Mark this reference.
- The reach shall be measured from the reference point on the floor to the point where the ball touches the platform.

**14.** Repeat data collection at least 5 times.

**15.** By pressing START the time measurement of the screen is sent to memory. For a new measure go back to item 10.

| N | A (m) | $\Delta t$ (s) | $v_{ox}$ (m/s) | $v_o$(m/s) |
|---|---|---|---|---|
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| 5 | | | | |
| | Average value | | | |

**>>> *Analysis of Results and Conclusions* >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>**

**16.** Use the follow equation and calculate the ball horizontal velocity ($v_{ox}$):

$$v_{ox} = \frac{A}{\Delta t}$$

**17.** Determine the average value of the launch velocity ($v_o$) for each measure carried out.

$$v_o = \frac{v_{ox}}{cos60^o}$$

**18.** Determine the average value of the launch velocity ($v_{om}$).

**19.** Compare the experimental values of the launch velocities found by processes (a) and (b). Was there any significant difference between the values found?

| *Method* | *By the time of passage between two sensors ($v_{o2}$)* | *By the residence time in the air ($v_{o3}$)* |
|---|---|---|
| *Velocity (m/s)* | | |

**20.** Compare the results.

# EXPERIMENT 03 – RELATIONSHIP BETWEEN REACH AND LAUNCH ANGLE

**Objective:**
**- Checking the dependence between a projectile's reach and launch angle.**

>>> ***Material Used*** >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

| Item | Code | Quant. | Unit | Description |
|---|---|---|---|---|
| 01 | 62002176 | 01 | Un. | CLAMP "C" |
| 02 | 62005751 | 01 | Un. | CANNON LAUNCHER HOLDER |
| 03 | 62002015 | 01 | Un. | CANNON (PROJECTILE LAUNCHER) |
| 05 | 62005120 | 01 | Un. | PLASTIC BALL Ø25MM |
| 06 | 62005177 | 01 | Un. | TUBE FOR CANNON COMPRESSION |
| 07 | 48005003 | 02 | Un. | BUTTERFLY NUT (CANNON'S FASTENER) |
| 08 | 50001004 | 02 | Un. | FLAT WASHER (CANNON'S FASTENER) |
| 09 | 62005317 | 01 | Un. | CANNON'S FASTENER |
| 10 | 62001023 | 01 | Un. | SENSORS/SPHERE SHOCK'S HOLDER |
| 13 | 03003011 | 01 | Un. | TAPE MEASURE 05M |
| 14 | 62005274 | 01 | Un. | PLUMB BOB WITH MAGNETIC FIXING |

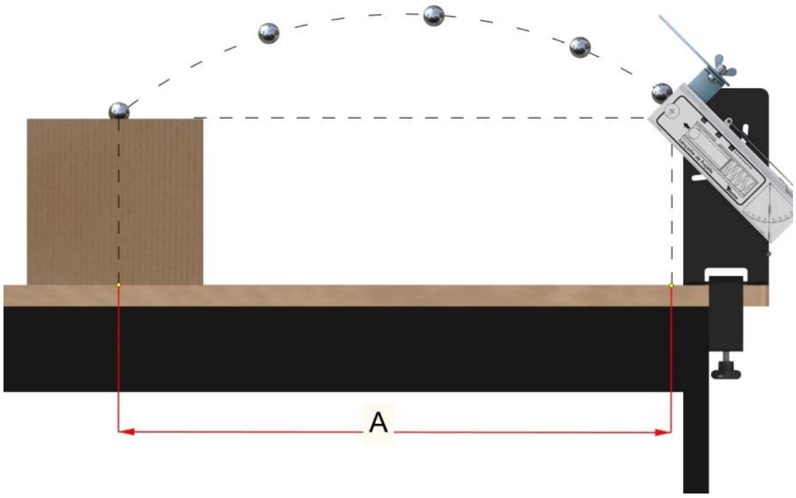
**(*) It does not accompany the product. It is sold separately.**

## Part I – Launch without gap

>>> ***Experimental Procedures***>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

Do not look directly at the cannon exit, as it may be charged!

**1.** Assemble the launcher as shown.




**2.** Provide a platform (for example, a box) that allows the projectile to reach it at the same horizontal point as the launch point.

**3.** Adjust the launch angle to 10º and insert the plastic ball into the second stage.

**4.** Make some previous launches to locate the point where the ball will touch the surface of the platform.

**5.** Fix a paper sheet on the platform and a carbon paper over

**6.** Make three launches and measure the value of launch position A (m) to the point where the ball touches the platform.

**7.** Repeat the launch procedures for the angles suggested in table 1.

Table 1

| Angle | Reach 1 | Reach 2 | Reach 3 | Reach measured | Reach calculated |
|---|---|---|---|---|---|
| 10 | | | | | |
| 20 | | | | | |
| 30 | | | | | |
| 40 | | | | | |
| 45 | | | | | |
| 50 | | | | | |
| 60 | | | | | |
| 70 | | | | | |
| 80 | | | | | |

>>> ***Analysis of Results and Conclusions*** >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

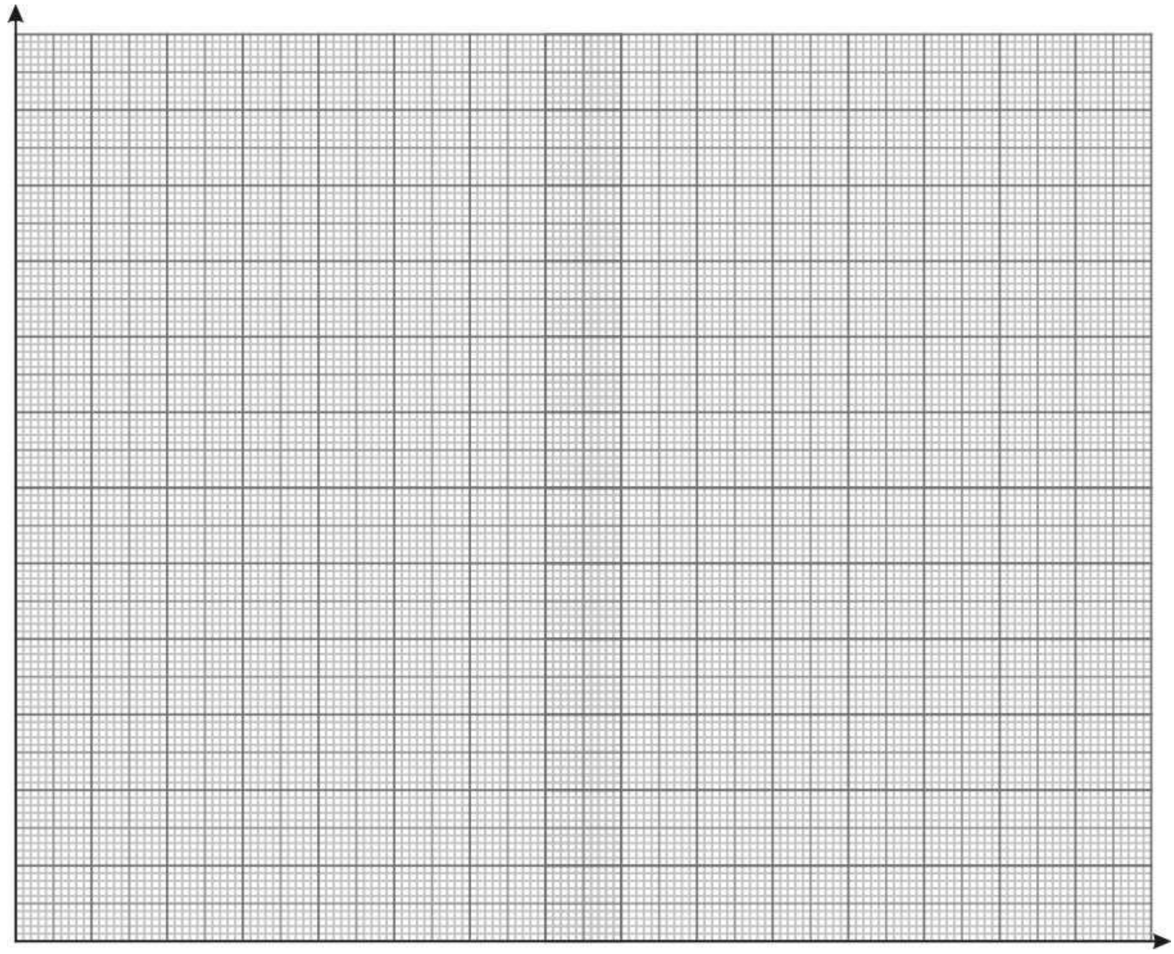
**8.** Use the experimental values from table 1 and calculate the average reach distance for each launch angle.

**9.** Use launch velocity equal to 4.12 in expression:

$$A_{calc} = \frac{v_0^2 \times sen2\theta}{g}$$

And obtain the calculated reach value for each angle.

**10.** Draw the reaches graph ($A_{measured}$) according to the launch angle.



**11.** According to the table and the graph, what is the launch angle value that provides the longest reach?

**12.** What can be concluded about the reach value for the complementary angles, that is, 10º and 80º, 20º and 70º, 30º and 60º, 40º and 50º?

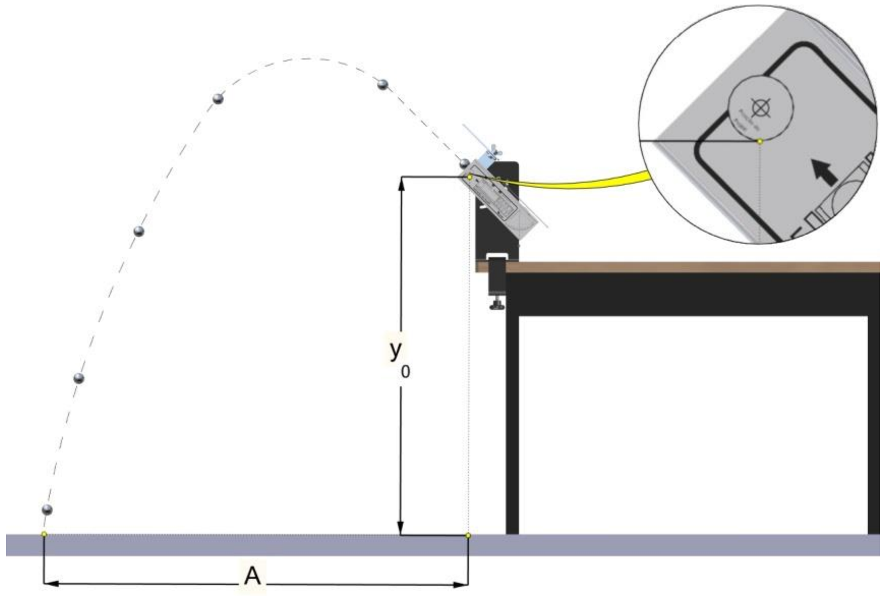
**13.** Are the experimental results consistent with the theory?

# Part II – Launch with gap

>>> ***Experimental Procedures***>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

Do not look directly at the cannon exit, as it may be charged!

**1.** Assemble the launcher as shown so that the projectile reaches the floor.




**2.** Use a plumb bob to determine the initial launch position on the floor.

**3.** Adjust the launch angle to 10º and insert the plastic ball into the second stage.

**4.** Fire at least three launches and measure the value of launch position A (m) to the point where the ball touches the floor. Fix a sheet of white paper on the platform and onto it a sheet of carbon paper.

**5.** Fire at least three launches and measure the value of launch position A (m) to the point where the ball touches the floor.

**6.** Repeat the launch procedures for the angles suggested in table 2.

Table 2

| Angle | Reach 1 (m) | Reach 2 (m) | Reach 3 (m) | $A_{average}$ |
|---|---|---|---|---|
| 10° | | | | |
| 20° | | | | |
| 30° | | | | |
| 40° | | | | |
| 50° | | | | |
| 60° | | | | |
| 70° | | | | |
| 80° | | | | |

>>> ***Analysis of Results and Conclusions*** >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

**7.** Use the experimental values from table 2 and calculate the average reach value ($A_{medido}$) for each launch angle.

**8.** Use the equation:

$$A_{calculado} = v_0 \cdot cos\theta \cdot \frac{v_0 \cdot sen\theta + \sqrt{v_0^2 \cdot sen^2\theta + 2gy_0}}{g}$$

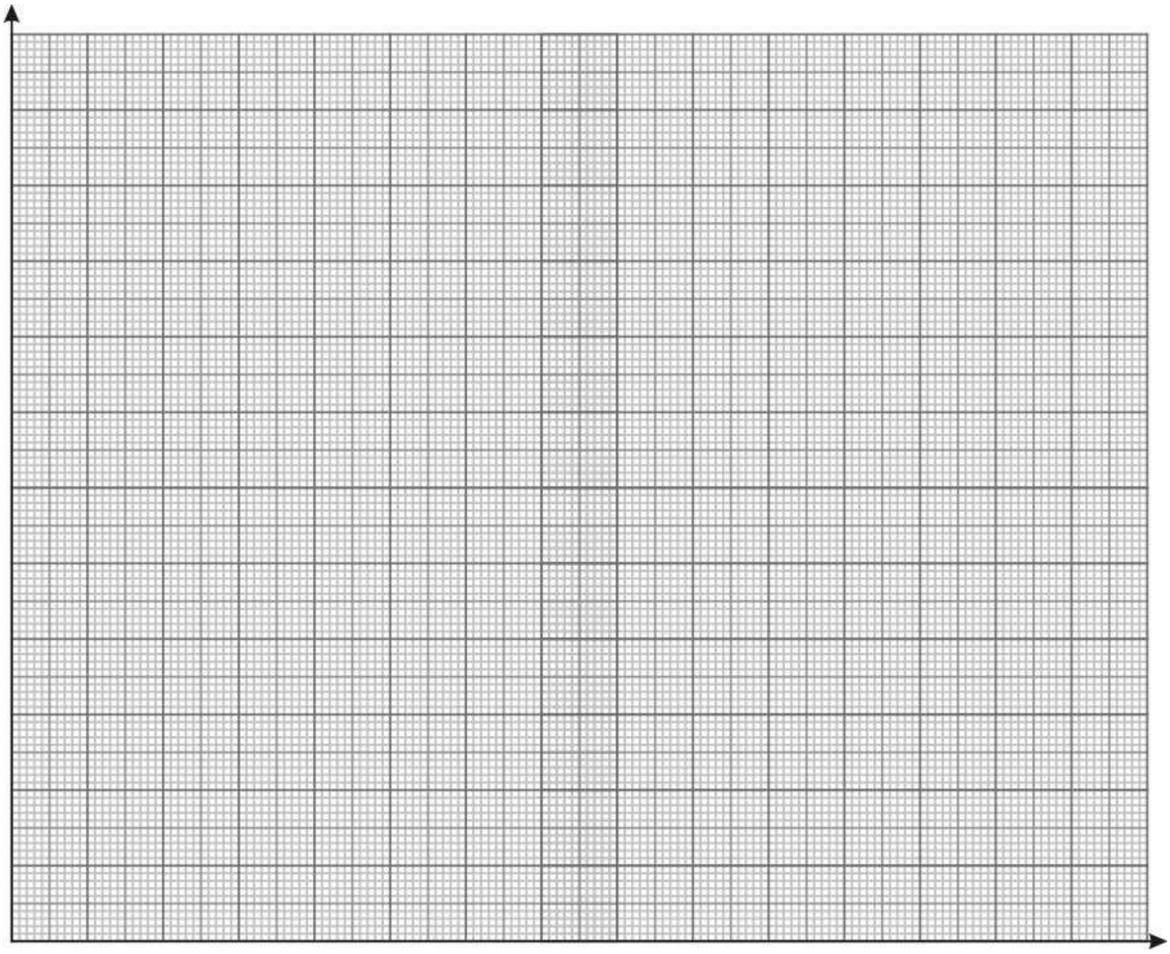
to obtain the reach calculated value

Table 3

| Angle | $A_{calculated}$ (m) | $A_{measured}$ (m) |
|---|---|---|
| $10^0$ | | |
| $20^0$ | | |
| $30^0$ | | |
| $40^0$ | | |
| $50^0$ | | |
| $60^0$ | | |
| $70^0$ | | |
| $80^0$ | | |

**9.** For each of the launch angles, are the reach values (measured and calculated) compatible?

**10.** Draw the reaches graph ($A_{measured}$) according to the launch angle.



**11.** According to the table and the graph, what is the launch angle value that provides the longest reach?

**12.** Are the measured and calculated values in agreement with the maximum reach value?

# EXPERIMENT 04 – ANALYSIS OF A TRAJECTORY OF A PROJECTILE

**Objective:**
**- Obtaining the relationship between the vertical and horizontal position of the projectile during its motion.**

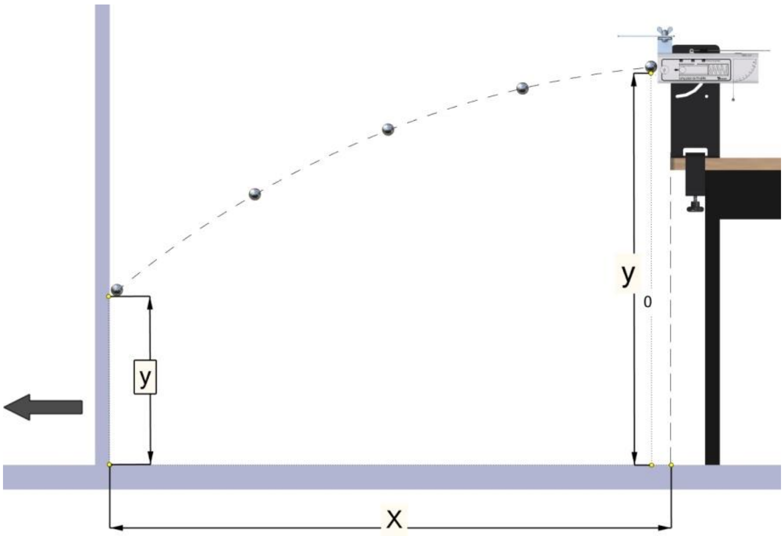
***>>> Material Used >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>***

| *Item* | *Code* | *Quant.* | *Unit* | *Description* |
|---|---|---|---|---|
| *01* | *62002176* | *01* | *Un.* | *CLAMP "C"* |
| *02* | *62005751* | *01* | *Un.* | *CANNON LAUNCHER HOLDER* |
| *03* | *62002015* | *01* | *Un.* | *CANNON (PROJECTILE LAUNCHER)* |
| *05* | *62001074* | *01* | *Un.* | *STEEL BALL Ø25MM* |
| *06* | *62005177* | *01* | *Un.* | *TUBE FOR CANNON COMPRESSION* |
| *07* | *48005003* | *02* | *Un.* | *BUTTERFLY NUT (CANNON'S FASTENER)* |
| *08* | *50001004* | *02* | *Un.* | *FLAT WASHER (CANNON'S FASTENER)* |
| *09* | *62005317* | *01* | *Un.* | *CANNON'S FASTENER* |
| *10* | *62001023* | *01* | *Un.* | *SENSORS/SPHERE SHOCK'S HOLDER* |
| *13* | *03003011* | *01* | *Un.* | *TAPE MEASURE 05M* |
| *14* | *62005274* | *01* | *Un.* | *PLUMB BOB WITH MAGNETIC FIXING* |

## Part I – Horizontal Launch

Do not look directly at the cannon exit, as it may be charged!

***>>> Experimental Procedures>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>***




**1.** Assemble the launcher as shown.

**2.** Adjust the cannon for horizontal launches. Use a plumb bob to determine on the floor the horizontal position of the projectile ($x_0 = 0$).

**3.** Assemble near a wall or set up a vertical bulkhead so that the ball, once launched, collides with it and determines the vertical and horizontal positions of the projectile in the collision with this target, as shown in the figure.

**4.** Position the bulkhead at x = 0.200 m from the horizontal launch position marked on the floor.

**5.** Fix a paper sheet on the target place on the wall and with a carbon paper over it.

**6.** Fire the ball and measure the value of the vertical position in which the projectile hits the target.

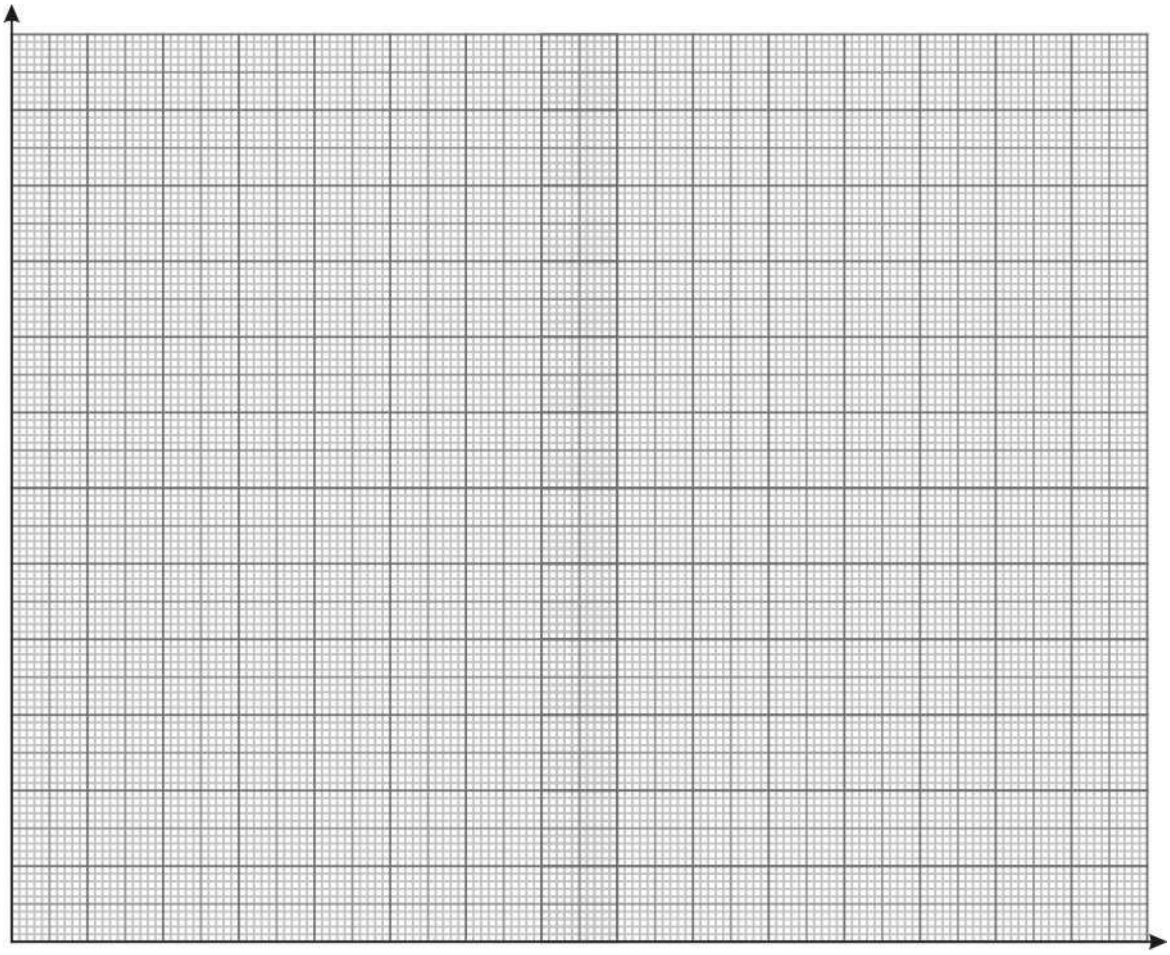
**7.** Reposition the target to the horizontal positions suggested in the table and repeat the launch procedures until you complete it. Complete the table 1.

Table 1

| X (m) | Y (m) | $X^2$ ($m^2$) |
|---|---|---|
| 0,000 | | |
| 0,200 | | |
| 0,400 | | |
| 0,600 | | |
| 0,800 | | |
| 1,000 | | |
| 1,200 | | |
| 1,400 | | |
| 1,600 | | |
| 1,800 | | |

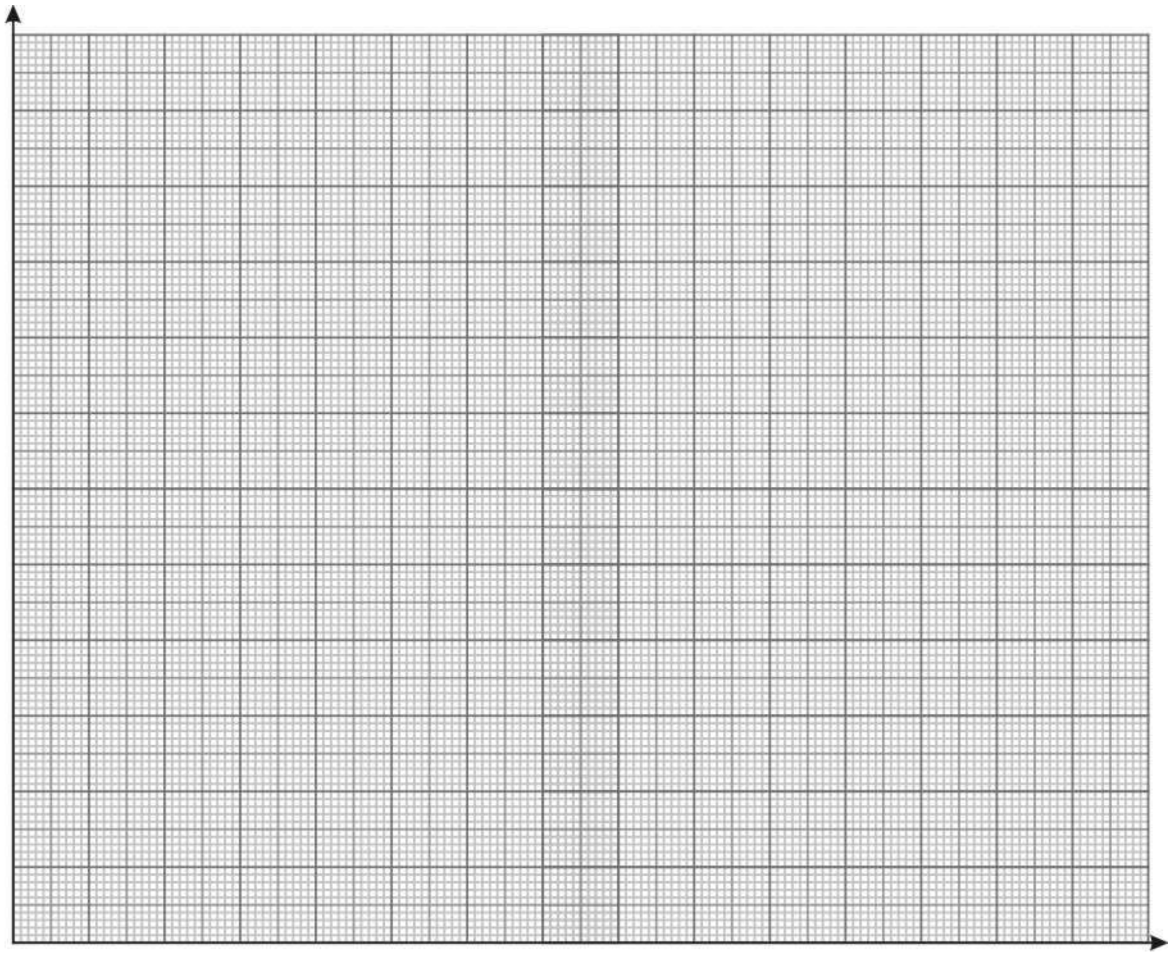
### >>> *Analysis of Results and Conclusions* >>>>>>>>>>>>>>>>>>>>>>>>>>>>

**8.** Use the values collected in table 2 and draw the graph "vertical position (y)" versus "horizontal position (x)".



**9.** What is the aspect of the curve obtained?

**10.** Do the appropriate change of variables and linearize the graph.



**11.** Obtain the equation that relates the quantities y x

**12.** Combine the equations of the two movements performed by the ball and obtain analytically the equation that relates the two positions (y and x) of the ball in its trajectory.

# Part II – Oblique launch

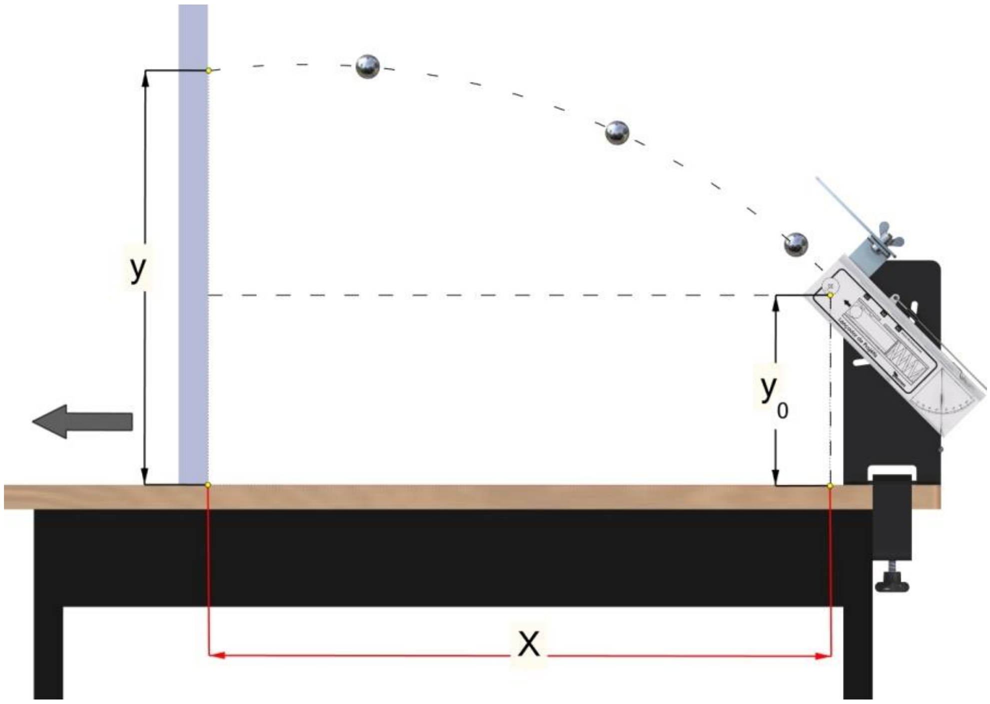
>>> ***Experimental Procedures***>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

Do not look directly at the cannon exit, as it may be charged!

**1.** Assemble the launcher as shown.

**2.** Adjust the launch angle by 60º and measure the height (yo) of the launch point in relation to the tabletop.

**3.** Assemble on the table a vertical bulkhead so that the ball, once launched, collides with it and determines the vertical and horizontal positions of the projectile in the collision with this target, as shown in the figure.




**4.** Use a plumb bob to obtain, on the tabletop, the initial horizontal position ($x_o$=0).

**5.** Use the steel ball and insert it into the second stage of the cannon.

**6.** Position the bulkhead at 0,050 m from the horizontal launch position ($x_o$ = 0) marked on the table top.

**7.** Affix a paper sheet on the target place with a carbon paper over it.

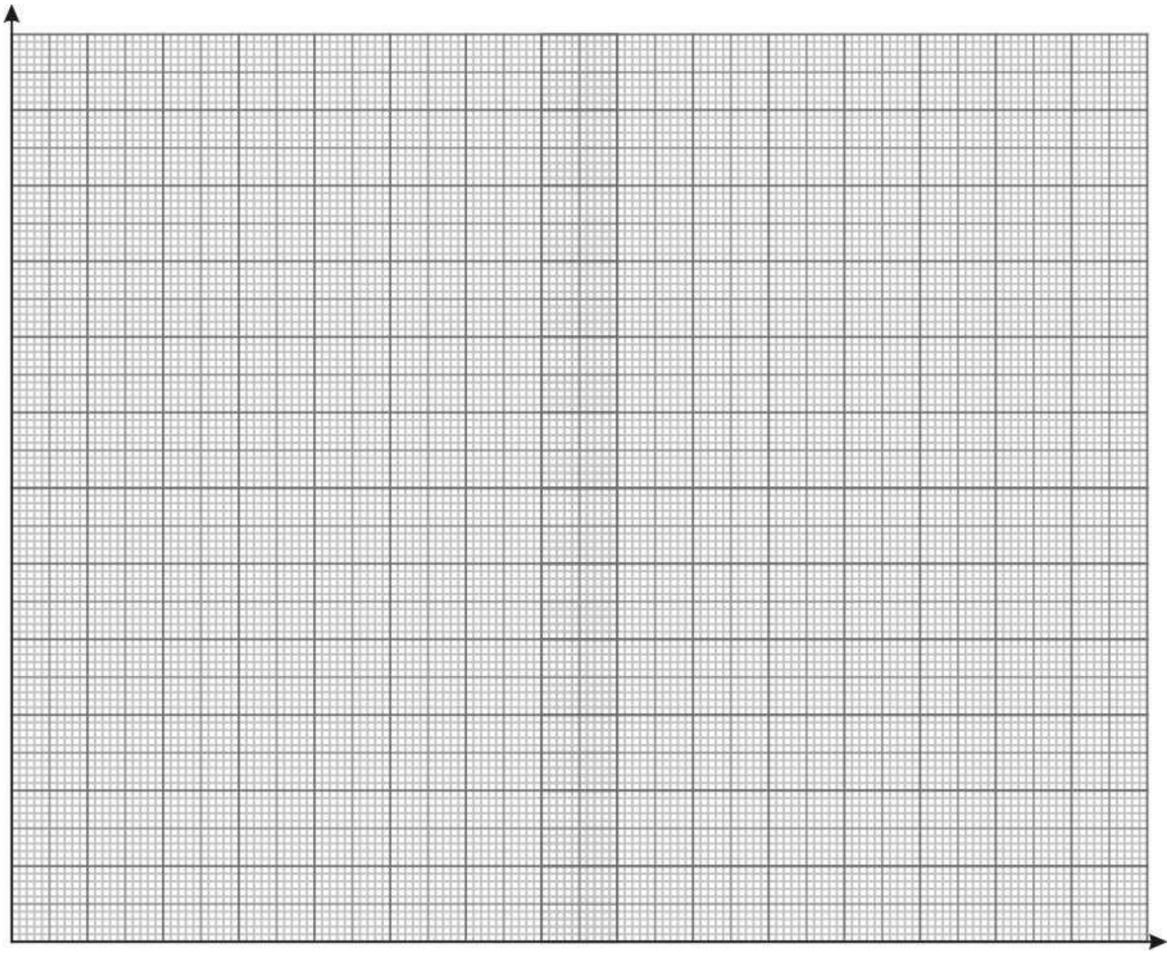
**8.** Reposition the target to the horizontal positions suggested in the table and repeat the launch procedures until you complete it. Complete the table 1.

Table 1

| N | x (m) | y (m) |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| 7 | | |
| 8 | | |
| 9 | | |
| 10 | | |
| 11 | | |
| 12 | | |

## >>> *Analysis of Results and Conclusions* >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

**9.** Draw the graph "vertical position y" versus "horizontal position x".



**10.** Use the software feature (Excel) and obtain the equation corresponding to the curve.

**11.** Combine the equations of the motions (vertical and horizontal) and obtain the equation y = f(x).

**12.** Is the dependence between y and x theoretically found consistent with the equation graphically found?

# EXPERIMENT 05 – ANALYSIS OF A TRAJECTORY OF A PROJECTILE II

**Objective:**
**- Verifying the validity of the equation that gives the projectile position at a given moment in an oblique launch.**

### >>> *Material Used* >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

| Item | Code | Quant. | Unit | Description |
|---|---|---|---|---|
| 01 | 62002176 | 01 | Un. | CLAMP "C" |
| 02 | 62005751 | 01 | Un. | CANNON LAUNCHER HOLDER |
| 03 | 62002015 | 01 | Un. | CANNON (PROJECTILE LAUNCHER) |
| 05 | 62005120 | 01 | Un. | PLASTIC BALL Ø25MM |
| 06 | 62005177 | 01 | Un. | TUBE FOR CANNON COMPRESSION |
| 07 | 48005003 | 02 | Un. | BUTTERFLY NUT (CANNON'S FASTENER) |
| 08 | 50001004 | 02 | Un. | FLAT WASHER (CANNON'S FASTENER) |
| 09 | 62005317 | 01 | Un. | CANNON'S FASTENER |
| 10 | 62001023 | 01 | Un. | SENSORS/SPHERE SHOCK'S HOLDER |
| 13 | 03003011 | 01 | Un. | TAPE MEASURE 05M |
| 14 | 62005274 | 01 | Un. | PLUMB BOB WITH MAGNETIC FIXING |
| XX | 62001226 | 01 | Un. | DIGITAL TIMER AZB-30 USB (*) |
| XX | 62001201 | 01 | Un. | PHOTOELECTRIC SENSOR PGS-D10 (*) |
| XX | 04002037 | 01 | Un. | FLIGHT TIME SENSOR TFS-D10 |
| XX | 29003002 | 01 | Un. | BIG TRIPOD (*) |
| XX | 30002014 | 01 | Un. | PAIR OF ADAPTABLE STEMS 12,7MM X 400MM (*) |
| XX | 31003001 | 01 | Un. | METALLIC CLIP AZB-027 |

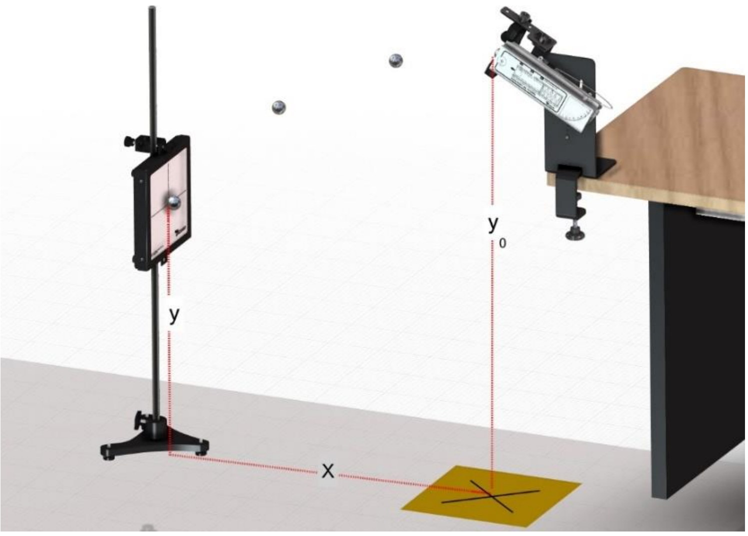
**(*) It does not accompany the product. It is sold separately.**

### >>> *Experimental Procedures*>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

Do not look directly at the cannon exit, as it may be charged!

**1.** Assemble the launcher at an angle of 40 ° to the horizontal on the side of the tabletop, so the projectile has space to move and fall on the floor, as shown in the figure.

**2.** Assemble the flight time sensor as shown.




**3.** Using the plumb bob, affix a paper sheet on the floor aligning with the cannon mouth.

**4.** Measure the launch height (yo) in relation to the floor. The measurement shall be made from the center of the ball indicated at the exit of the cannon, to the floor.

**5.** Position the flight time sensor at x = 0.400m from the launcher's mouth. Cover the face of the sensor with a carbon paper.

**6.** Place the plastic ball in the projectile launcher and compress the spring to the first stage. Pull the trigger and measure the height y (vertical position) in which the ball touches the platform. Repeat this procedure three times and note in the table the average value of y.

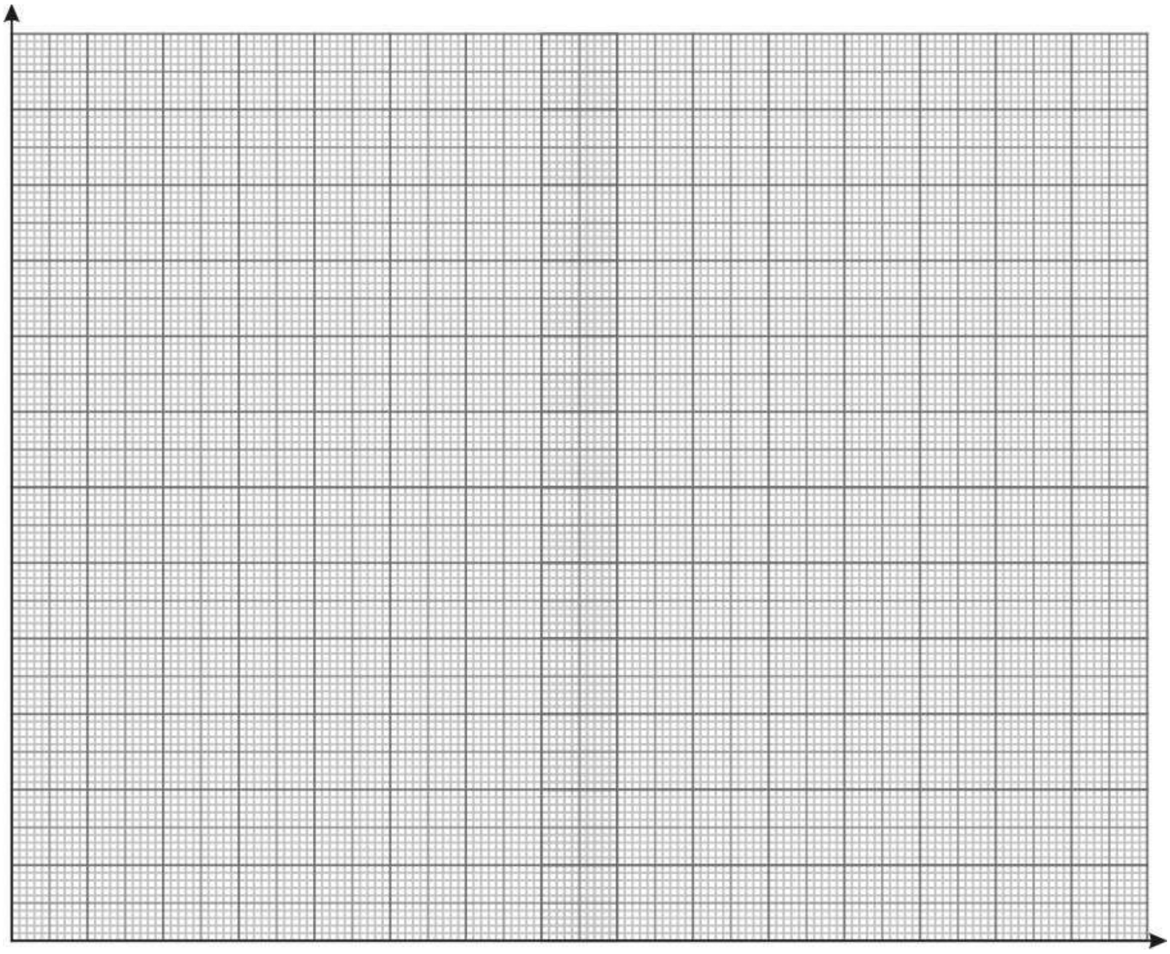
**7.** Reposition the flight time sensor to the positions suggested in the table and repeat the experimental procedures to obtain the values of y.

| N | Horizontal position x (m) | Vertical position y (m) |
|---|---|---|
| 1 | 0,000 | |
| 2 | 0,200 | |
| 3 | 0,400 | |
| 4 | 0,600 | |
| 5 | 0,800 | |
| 6 | 1,000 | |
| 7 | 1,200 | |
| 8 | 1,400 | |
| 9 | 1,600 | |
| 10 | 1,790 | |

**>>> *Analysis of Results and Conclusions* >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>**

**8.** Combine the oblique launch equations of a projectile and obtain the expression that gives the vertical position y according to the horizontal position x, that is: $y = f(x)$.

**9.** With the experimental data of y and x draw the graph y versus x.



**10.** What does the graph $y = f(x)$ look like?

**11.** Obtain the equation that represents the curve obtained in the graph.

**12.** Does the equation obtained experimentally agree with the theoretical expression?

# EXPERIMENT 06 – TRAJECTORY OF A PROJECTILE IN THE HORIZONTAL LAUNCH

**Objective: Verifying if a horizontally launch projectile describes a parabolic trajectory.**

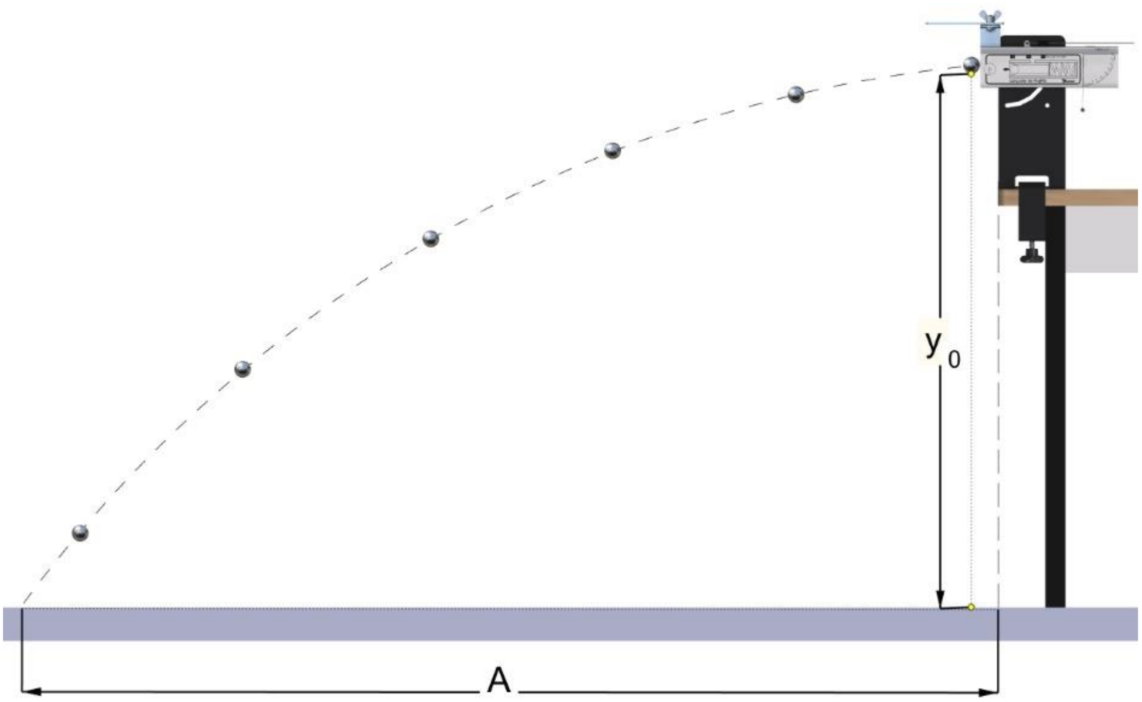
>>> ***Material Used*** >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

| Item | Code | Quant. | Unit | Description |
|---|---|---|---|---|
| 01 | 62002176 | 01 | Un. | CLAMP "C" |
| 02 | 62005751 | 01 | Un. | CANNON LAUNCHER HOLDER |
| 03 | 62002015 | 01 | Un. | CANNON (PROJECTILE LAUNCHER) |
| 05 | 62005120 | 01 | Un. | PLASTIC BALL Ø25MM |
| 06 | 62005177 | 01 | Un. | TUBE FOR CANNON COMPRESSION |
| 07 | 48005003 | 02 | Un. | BUTTERFLY NUT (CANNON'S FASTENER) |
| 08 | 50001004 | 02 | Un. | FLAT WASHER (CANNON'S FASTENER) |
| 09 | 62005317 | 01 | Un. | CANNON'S FASTENER |
| 10 | 62001023 | 01 | Un. | SENSORS/SPHERE SHOCK'S HOLDER |
| 13 | 03003011 | 01 | Un. | TAPE MEASURE 05M |
| 14 | 62005274 | 01 | Un. | PLUMB BOB WITH MAGNETIC FIXING |
| XX | 29003002 | 04 | Un. | BIG TRIPOD (*) |
| XX | 30002014 | 06 | Un. | PAIR OF ADAPTABLE STEMS 12,7MM X 400MM (*) |
| XX | 02005006 | 04 | Un. | LAB CLAMP WITH RING (*) |
| XX | 31003001 | 04 | Un. | METALLIC CLIP AZB-02 (*) |

(*) It does not accompany the product. It is **s**old separately.

>>> ***Experimental Procedures***>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

**13.** Assemble the launcher as shown.




**1.** Measure the launch height (yo) in relation to the floor. The measurement shall be made from the lower part of the ball indicated at the exit of the cannon, to the floor, according to the figure.

**2.** Affix a paper sheet and mark the vertical alignment of the plumb with the center of the ball, as shown.

**3.** Place the plastic ball in the cannon and compress the spring to the first stage. Pull the trigger and observe where the ball touches the floor.

**4.** At the dropping point of the ball, affix a paper sheet and on it a carbon paper, marking the projectile's reach distance A.

**5.** Repeat the launch 5 times and measure the reached distance A by using the measuring tape.

Table 1

| N | Reach (m) |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| Average value | |

>>> ***Analysis of Results and Conclusions*** >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

**6.** Determine the average reach distance.

**7.** By combining equations:

$$\begin{cases} A = v_0 \cdot t \\ y = y_0 - \dfrac{g \cdot t^2}{2} \end{cases}$$

We obtain the equation that provides the launch velocity:

$$v_0 = A \cdot \sqrt{\frac{g}{2y_0}}$$

**8.** Calculate the projectile horizontal launch velocity $v_0$ by using the reach average value.

**9.** Draw on the sheet of paper placed on the floor a line that joins the point marked with the plumb bob and the point corresponding to the reach average value.

**10.** Mark from the origin along the line drawn on the paper the following positions:

$x_1$ = 0,30 m; $x_2$ = 0,60 m; $x_3$ = 0,90 m; $x_4$ = 1,20 m.

**11.** The vertical motion equation is:

$$y = y_o - \frac{g}{2}t^2; \quad (v_{0y} = 0)$$

By combining with horizontal motion equation, $x = v_0 \cdot t$, results the equation that relates the projectile positions y and x, y=f(x):

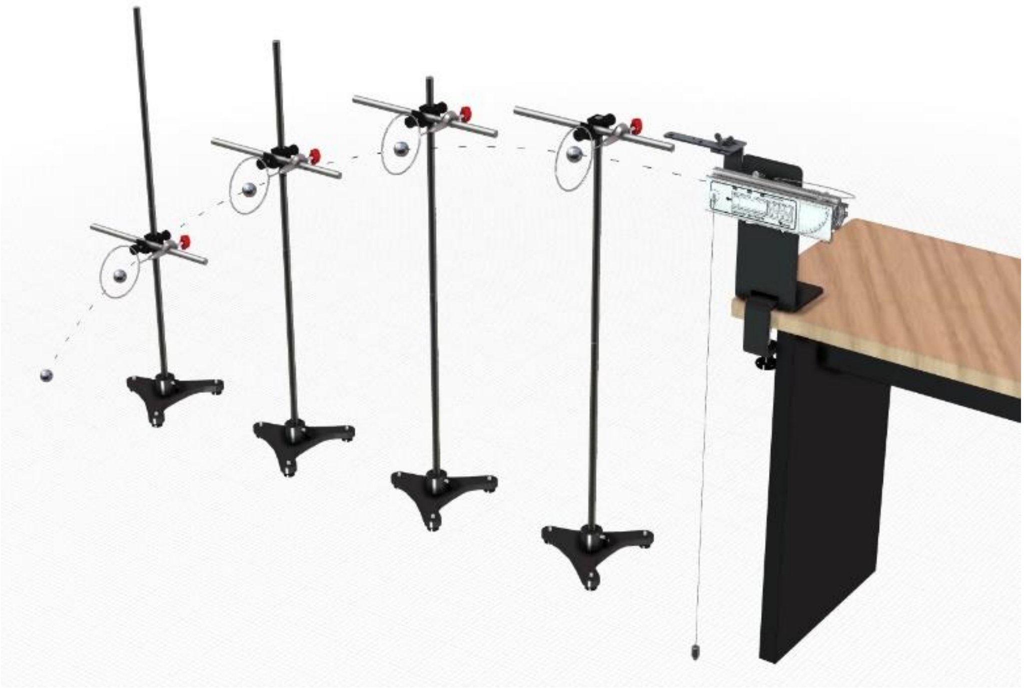
$$y = y_0 - \frac{g}{2v_0^2}x^2$$

**12.** Calculate the value of the vertical y position of the projectile for x positions marked on the sheet of paper and suggested in the table.

Table 2

| x (m) | $y_o$ (m) | $v_o$ (m/s) | y (m) |
|---|---|---|---|
| 0,300 | | | |
| 0,600 | | | |
| 0,900 | | | |
| 1,200 | | | |

**13.** Assemble four supports for the rings and position them in x positions suggested in table 2, as shown in the figure.



**14.** Prepare the projectile launch with the plastic ball in the first stage. Fire the launch and verify if the projectile has passed through the rings without touching them.

# EXPERIMENT 07 – CONSERVATION OF ENERGY

**Objective:**
**Verifying conservation of mechanical energy using the vertical launch.**

*>>> Material Used >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>*

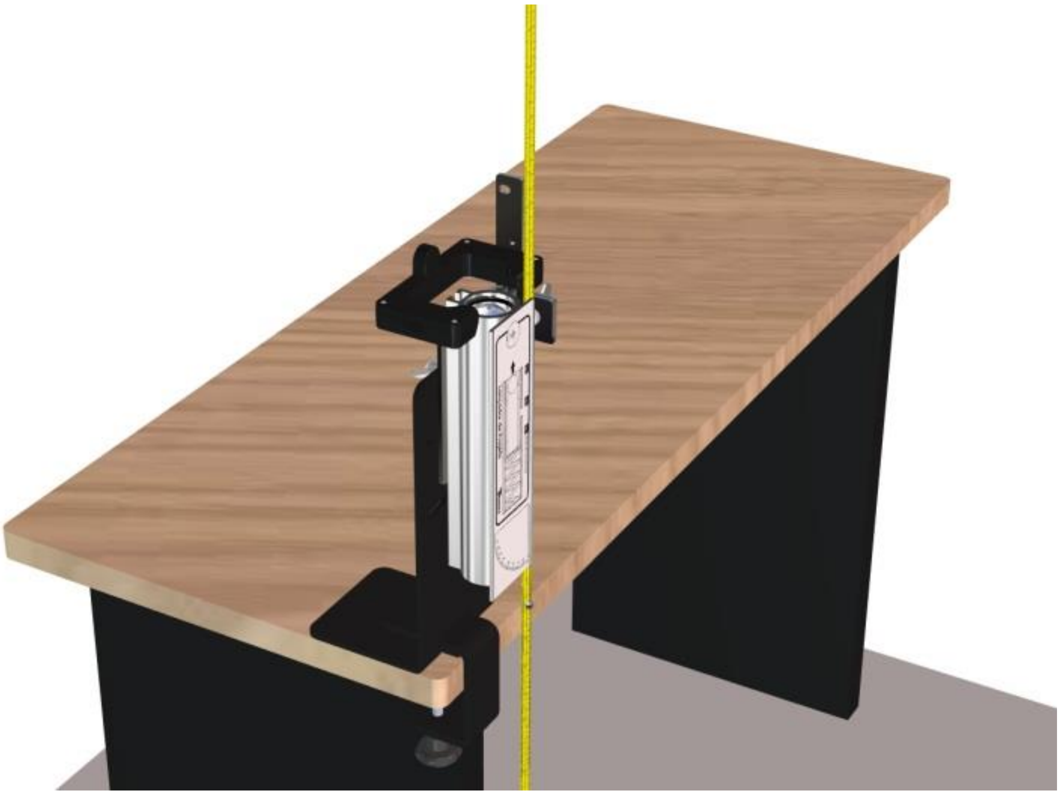
| Item | Code | Quant. | Unit | Description |
|---|---|---|---|---|
| 01 | 62002176 | 01 | Un. | CLAMP "C" |
| 02 | 62005751 | 01 | Un. | CANNON LAUNCHER HOLDER |
| 03 | 62002015 | 01 | Un. | CANNON (PROJECTILE LAUNCHER) |
| 04 | 62001074 | 01 | Un. | STEEL BALL Ø25MM |
| 06 | 62005177 | 01 | Un. | TUBE FOR CANNON COMPRESSION |
| 07 | 48005003 | 02 | Un. | BUTTERFLY NUT (CANNON'S FASTENER) |
| 08 | 50001004 | 02 | Un. | FLAT WASHER (CANNON'S FASTENER) |
| 09 | 62005317 | 01 | Un. | CANNON'S FASTENER |
| 10 | 62001023 | 01 | Un. | SENSORS/SPHERE SHOCK'S HOLDER |
| 13 | 03003011 | 01 | Un. | TAPE MEASURE 05M |
| 14 | 62005274 | 01 | Un. | PLUMB BOB WITH MAGNETIC FIXING |
| XX | 62001226 | 01 | Un. | DIGITAL TIMER AZB-30 USB (*) |
| XX | 62001201 | 01 | Un. | PHOTOELECTRIC SENSOR PGS-D10 (*) |

**(*) It does not accompany the product. It is sold separately.**

***>>> Experimental Procedures>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>***

Do not look directly at the cannon exit, as it may be charged!

**1.** Assemble the projectile launcher as shown.



**2.** Adjust the launch angle to 90º.

**3.** Measure the diameter (Ø) and the mass (m) of the steel ball.

**4.** Assemble a tripod with a stem on the table and fix an object (a ruler, for example) that serves as a reference to determine the vertical reach of the projectile, as shown in the figure.

**5.** Fix the sensor to measure the ball passage time, in the position shown in the figure.

**6.** Connect the timer and adjust it to use the function 2, with the screens shown below:



To select the desired measurement type, press the SETUP key for 2s to enter the configuration mode. In the configuration mode, use the START / RESET key to navigate among the configuration parameters. To change a selected parameter use the SETUP / MEM ↑ keys. After setting the function, press the FUNC key to save the selected parameters.

**7.** Position the steel ball in the second stage of the launcher and fire the cannon 5 times to determine the vertical reach value of the ball, adjusting the referential at each launch.

**8.** Note in table 1 the vertical reach and the time of passage values of the ball by the sensor.

Table 1

| N | y (m) | t(s) |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| $y_{average}$ | | |

**>>> *Analysis of Results and Conclusions* >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>**

**9.** Calculate the average value of the maximum height (y) reached by the ball.

**10.** Calculate the average value of the passage time (t).

**11.** Use the ball diameter value and the average passage time and calculate the launch velocity modulus $v_0$.

**12.** Calculate the kinetic energy value in the initial position (at the time of launch).

**13.** Calculate the value of the gravitational potential energy at the highest point of the path.

**14.** Compare the values of mechanical energy in both procedures and justify the discrepancy found:

**15.** Does the experiment confirm the principle of conservation of mechanical energy?

# EXPERIMENT 08 – CONSERVATION OF THE LINEAR MOMENT

**Objective:**
**Verifying the conservation of linear momentum in elastic and inelastic collision.**

**>>> *Material Used* >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>**

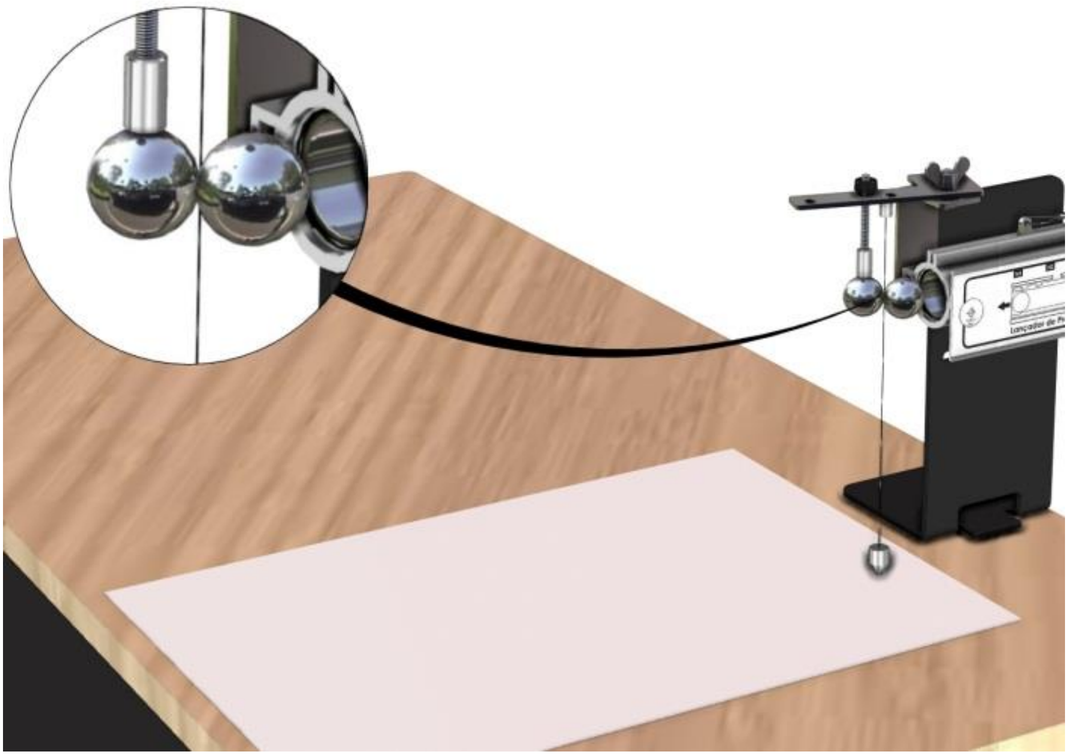
| Item | Code | Quant. | Unit | Description |
|---|---|---|---|---|
| 01 | 62002176 | 01 | Un. | CLAMP "C" |
| 02 | 62005751 | 01 | Un. | CANNON LAUNCHER HOLDER |
| 03 | 62002015 | 01 | Un. | CANNON (PROJECTILE LAUNCHER) |
| 04 | 62001074 | 02 | Un. | STEEL BALL Ø25MM |
| 06 | 62005177 | 01 | Un. | TUBE FOR CANNON COMPRESSION |
| 07 | 48005003 | 02 | Un. | BUTTERFLY NUT (CANNON'S FASTENER) |
| 08 | 50001004 | 02 | Un. | FLAT WASHER (CANNON'S FASTENER) |
| 09 | 62005317 | 01 | Un. | CANNON'S FASTENER |
| 10 | 62001023 | 01 | Un. | SENSORS/SPHERE SHOCK'S HOLDER |
| 11 | 62002056 | 01 | Un. | BALL'S MAGNETIC FIXING (FOR COLLISION) |
| 12 | 53003001 | 01 | Un. | FASTENER FOR BALL'S MAGNETIC FIXING |
| 13 | 03003011 | 01 | Un. | TAPE MEASURE 05M |
| 14 | 62005274 | 01 | Un. | PLUMB BOB WITH MAGNETIC FIXING |

## Part I – Elastic Oblique Collision

**>>> *Experimental Procedures*>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>**

Do not look directly at the cannon exit, as it may be charged!

**1.** Assemble the projectile launcher as shown.

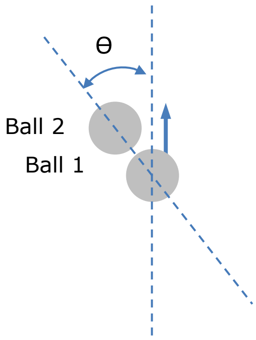


2. Position the launcher for horizontal launch
3. Measure the mass of each steel ball.

4. Attach the magnetic fixing as shown previously.

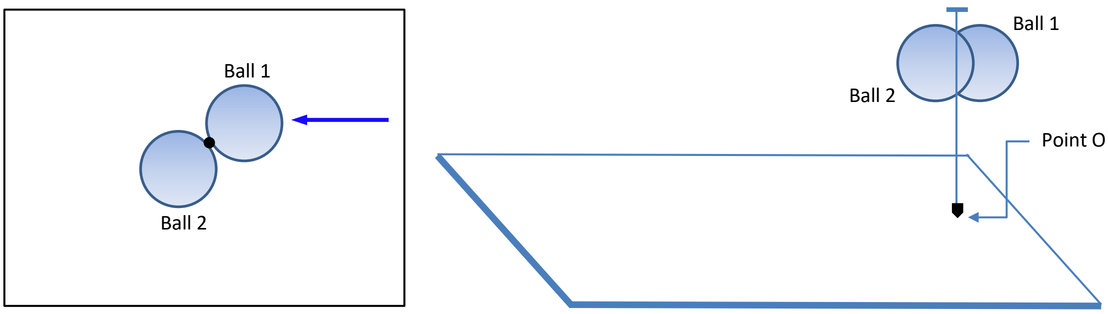
5. Fire the cannon once with the steel ball 1 in the second stage and adjust the position of the launcher to ensure that the ball falls on the sheet of paper covering the table.

6. Place the second steel ball (2) on the magnetic fixing and adjust its position very well for an oblique collision at an angle Θ around 40 ° to 50 °. It is advisable that the collision occurs at least 3.0 cm from the launcher's mouth.




7. Fire launches that produce a collision between steel balls 1 and 2 and make adjustments to the launcher and the support of ball 2 positions that ensure that in these procedures (with and without collision) the balls fall on the table.

8. Use a plumb bob and determine on the sheet of paper placed on the table the point O where the collision occurs. This point will be considered as the origin of horizontal displacement.




9. Measure the launch height $h_o$ in relation to the table surface.

10. Remove the ball 2 from the magnetic support.

11. Cover the sheet of paper with carbon paper in the estimated position.

12. Fire five launches with ball 1 in the first stage and determine the point where the ball touches the table surface.

**13.** Measure the reach value in each launch, note them in table 1 and determine the average value $A_o$.

Tale 1

| N | Height $h_O$ (m) | Reach $A_o$ (m) |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| Average value | | |

**14.** Place the ball 1 in the first stage of the launcher. Position the ball 2 on the magnetic fixing and fire the launcher. Observe where the balls touch the table surface after the collision. Place in these positions sheets of carbon paper.

**15.** Prepare a new launch for the two balls collision.

**16.** Fire the launcher and measure the two balls reaches $A_1$ and $A_2$ after the collision.

**17.** Repeat the launch process five times and determine the average position of each ball reach after the collision. Complete table 2

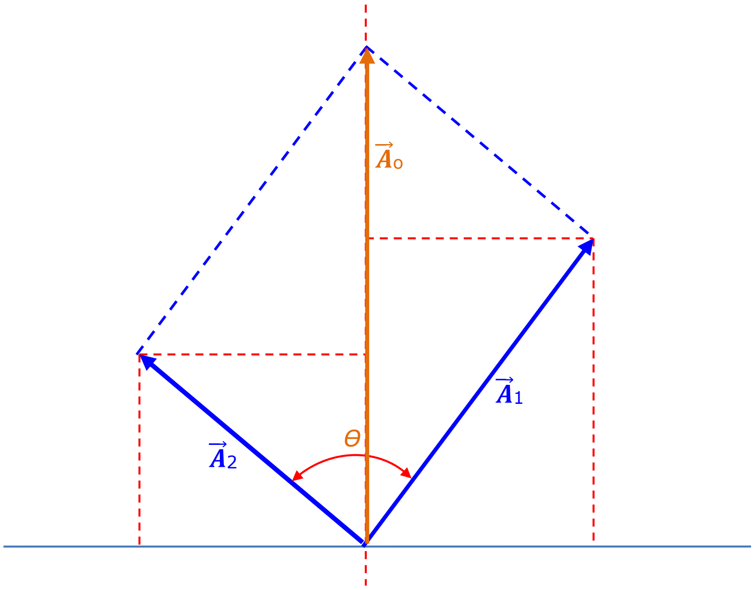
Table 2

| *event* | *$A_1$ (m)* | *$A_2$ (m)* |
|---|---|---|
| *1* | | |
| *2* | | |
| *3* | | |
| *4* | | |
| *5* | | |
| *Average value* | | |

**>>> *Analysis of Results and Conclusions* >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>**

**18.** Draw on the sheet of paper from the point O, the lines corresponding to the average reaches $A_0$ of the ball 1 before the collision, A1 of the ball1 and A2 of the ball 2, after the collision.

**19.** Measure the angle Θ between the vectors corresponding to A1 and A2.

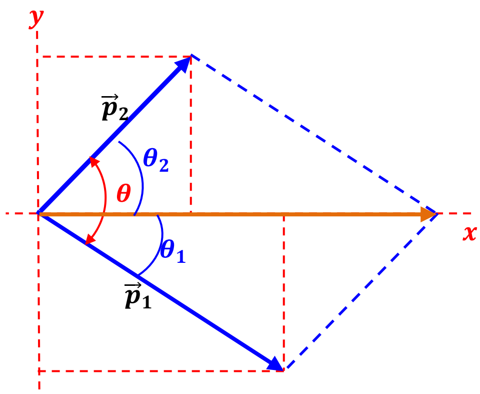



**20.** Use the average value of table 1 and the appropriate equations to calculate the initial momentum modulus $p_o$ of the system:

$$v_o = A \cdot \sqrt{\frac{g}{2h_o}} \text{e} p_o = m_{esf} \cdot v_o$$

**21.** Use the data in table 2 and calculate the values of linear momentum modules of each ball after the collision.

**22.** Scale the vectors $\vec{p}_o$, $\vec{p}_1$ and $\vec{p}_2$.




**23.** Apply the cosine law and obtain the vector module $\vec{p}$, resulting from the vectors $\vec{p}_1$ and $\vec{p}_2$.

**24.** Compare the values of the linear momentum modules just before and immediately after the collision. Has the linear momentum of the system been conserved?

Table 3

| *Initial linear momentum $p_o$ (kg.m/s)* | *Final linear momentum (kg.m/s)* | *Percent error* |
|---|---|---|
| | | |

**25.** Take the direction x of reference as the direction of the ball's reach before the collision and trace the pair of axes (x, y). Measure the angles that each vector forms with the x-axis.

**26.** Obtain the orthogonal components of the linear moments after the collision:

Table 4

| | *Module* | *Angle with x-axis* | *Component x $p_{ox}$* | *Component y $p_{oy}$* |
|---|---|---|---|---|
| $p_o$ | | | | |
| $p_1$ | | | | |
| $p_2$ | | | | |
| $p$ | | | | |

**27.** Can we consider that in the direction x the linear momentum has been conserved? And in the direction y?

**28.** Was there conservation of kinetic energy? Can collision be considered as an elastic collision? Justify.

Table 5

| Initial kinetic energy (J) | Final kinetic energy (J) | | | |
|---|---|---|---|---|
| | Ball 1 | Ball 2 | Sum | e% |
| | | | | |

# Part II – Inelastic Oblique Collision

>>> ***Experimental Procedures***>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

Do not look directly at the cannon exit, as it may be charged!

**1.** Use the same assembly of the first part with steel balls.

**2.** Consider the data from the table obtained with the ball 1 launch without collision.

Table 1

| N | Launch height $h_0$ (m) | Reach $A_0$ (m) |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| Average value | | |

**3.** Adopt as axis x the launch direction without collision.

**4.** Wrap the ball 2 with one lap of sticky tape and repeat the collision launch procedures. In this case it is advisable to Fire only one launch, since the replacement of the ball 2 would hardly occur in the same positioning conditions

**5.** Lay on the table a sheet of paper where the balls fall after the collision.

**6.** Repeat the procedures for marking the point of origin of the horizontal displacements of the balls.

**7.** Fire some previous launches and observe the points where the balls touch the table. Put sheets of carbon paper at these points.

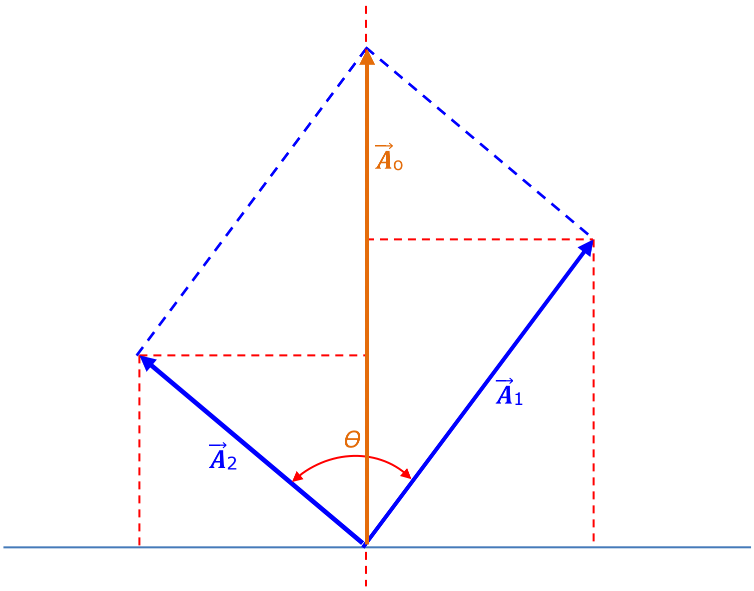
**8.** Fire the cannon and score the points obtained from the two balls reaches.

| | Reach (m) | Angle with axis x |
|---|---|---|
| Ball 1 | | |
| Ball 2 | | |

>>> ***Analysis of Results and Conclusions*** >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

**9.** Calculate the value of the linear momentum module of the system immediately before the collision.

**10.** Draw on the sheet of paper from the point O, the lines corresponding to the average reaches A0 of the ball 1 before the collision, A1 of the ball1 and A2 of the ball 2, after the collision.




**11.** Measure the angle that each vector forms with the vector direction obtained without the collision.

**12.** Measure the angle Θ between the vectors corresponding to A1 and A2.

**13.** Calculate the value of the velocity and momentum modules of each ball after the collision. The launch height $h_0$ is the same as the first part. ($h_o$ =0,189 m).

$$v = A \cdot \sqrt{\frac{g}{2h_o}} \text{e} p_o = m_{esf} \cdot v_o$$

**14.** Use the cosine law and calculate the value of the linear momentum module after the collision.

**15.** Was there conservation of the linear momentum of the system in the inelastic collision?

**16.** Calculate the initial and final kinetic energy of the system. Was there energy conservation in the collision?

# Part III – Frontal Elastic Collision

>>> ***Experimental Procedures***>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

Do not look directly at the cannon exit, as it may be charged!

**1.** Use the same assembly as in the first part.

**2.** Consider the data of table 1 obtained with the ball 1 launch without collision.

Table 1

| N | Launch height $h_O$ (m) | Reach $A_o$ (m) |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| Average value | | |

**3.** Adopt as x-axis the launch direction without collision.

**4.** Adjust the position of the ball 2 support so the centers of the two balls are on the same axis providing a frontal collision.

**5.** Lay on the table a sheet of paper to accommodate the balls after the collision.

**6.** Fire the cannon a couple of times and adjust the position of ball 2 until a really frontal collision (without lateral deviation of the balls after the collision).

**7.** Cover the sheet with carbon paper in the regions where the balls touch the table.

**8.** Trigger the launcher and mark the reaches' points A1 and A2 of the two balls.

***>>> Analysis of Results and Conclusions*** >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

**9.** Draw the lines corresponding to the scales, measure and note the value of the reaches modules $A_1$ and $A_2$ of each ball.

**10.** Measure the angles $\Theta_1$ and $\Theta_2$ which $A_1$ and $A_2$ form with the x-axis and the angle $\Theta$ between $A_1$ and $A_2$.

Table 2

| | Reach (m) | Angle with x-axis |
|---|---|---|
| Ball 1 | | |
| Ball 2 | | |

**11.** Calculate the value of the velocity and momentum modules of each ball after the collision. The launch height $h_0$ is the same as the first part. ($h_o$ =0,189 m).

$$v_o = A \cdot \sqrt{\frac{g}{2h_o}} \text{e} p_o = m_{esf} \cdot v_o$$

**12.** Obtain the linear momentum module after the collision.

**13.** Was there conservation of the linear momentum of the system in the inelastic collision?

**14.** Calculate the initial and final kinetic energy of the system. Was there energy conservation in the collision?

# Part IV – Frontal Inelastic Collision

**>>> *Experimental Procedures*>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>**

Do not look directly at the cannon exit, as it may be charged!

**1.** Use the same assembly as in the first part.

**2.** Consider the data of table 1 obtained with the ball 1 launch without collision.

Table 1

| N | Launch height $h_O$ (m) | Reach $A_o$ (m) |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| Average value | | |

**3.** Adopt as x-axis the launch direction without collision.

**4.** Wrap ball 2 with one lap of sticky tape and repeat collision launch procedures. In this case it is advisable to fire only one time, since the replacement of the ball 2 would hardly occur in the same positioning conditions.

**5.** Adjust the position of the ball 2 support so the centers of the two balls are on the same axis providing a frontal collision.

**6.** Lay on the table a sheet of paper to accommodate the balls after the collision.

**7.** Fire a couple of times and adjust the position of ball 2 until a really frontal collision (without lateral deviation of the balls after the collision).

**8.** Cover the sheet with carbon paper in the regions where the balls touch the table.

**9.** Fire the launcher and mark the reaches' points A1 and A2 of the two balls.

**>>> *Analysis of Results and Conclusions* >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>**

**10.** Draw the lines corresponding to the scales, measure and note the value of the reaches modules A1 and A2 of each ball.

**11.** Measure the angles $\Theta_1$ and $\Theta_2$ which $A_1$ and $A_2$ form with the x-axis and the angle $\Theta$ between $A_1$ and $A_2$.

Table 2

| | Reach (m) | Angle with x-axis |
|---|---|---|
| Ball 1 | | |
| Ball 2 | | |

**12.** Calculate the value of the velocity and momentum modules of each ball after the collision. The launch height $h_0$ is the same as the first part. ($h_o$ =0,189 m).

$$v_o = A \cdot \sqrt{\frac{g}{2h_o}} \text{e} p_o = m_{esf} \cdot v_o$$

**13.** Obtain the linear momentum module after the collision.

**14.** Was there conservation of the linear momentum of the system in the inelastic collision?

**15.** Calculate the initial and final kinetic energy of the system. Was there energy conservation in the collision?

# EXPERIMENT 09 – LAUNCH ANGLE THAT MAXIMIZES THE HEIGHT OF A PROJECTILE

**Objective:**
**- Finding the launch angle that maximizes the height reached by projectile launched from a fixed distance from a bulkhead.**

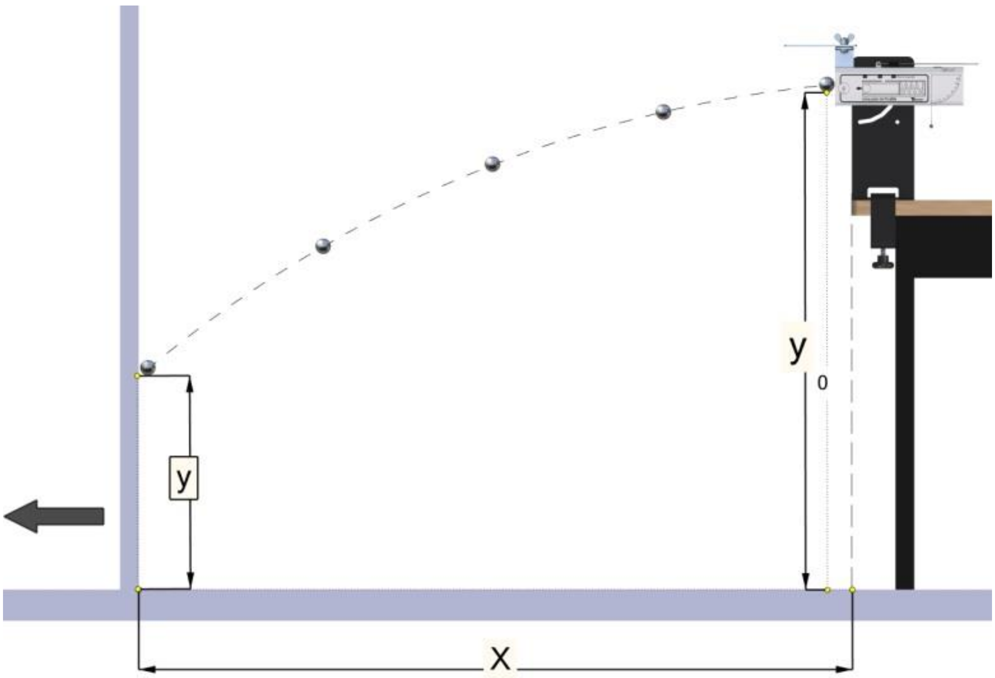
***>>> Material Used >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>***

| *Item* | *Code* | *Quant.* | *Unit* | *Description* |
|---|---|---|---|---|
| *01* | *62002176* | *01* | *Un.* | *CLAMP "C"* |
| *02* | *62005751* | *01* | *Un.* | *CANNON LAUNCHER HOLDER* |
| *03* | *62002015* | *01* | *Un.* | *CANNON (PROJECTILE LAUNCHER)* |
| *05* | *62005120* | *01* | *Un.* | *PLASTIC BALL Ø25MM* |
| *06* | *62005177* | *01* | *Un.* | *TUBE FOR CANNON COMPRESSION* |
| *07* | *48005003* | *02* | *Un.* | *BUTTERFLY NUT (CANNON'S FASTENER)* |
| *08* | *50001004* | *02* | *Un.* | *FLAT WASHER (CANNON'S FASTENER)* |
| *09* | *62005317* | *01* | *Un.* | *CANNON'S FASTENER* |
| *13* | *03003011* | *01* | *Un.* | *TAPE MEASURE 05M* |
| *14* | *62005274* | *01* | *Un.* | *PLUMB BOB WITH MAGNETIC FIXING* |

***>>> Experimental Procedures>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>***

Do not look directly at the cannon exit, as it may be charged!

**1.** Assemble the launcher as shown.




**2.** Position the table facing a wall so the launcher's mouth is about 1.60 m away. Use a plumb bob to determine the initial launch position on the floor and measure the distance (x) from the wall to the launcher's mouth and the vertical launch position ($y_o$).

**3.** Fire the cannon and determine where to place a 2 sheet of paper, displayed vertically.

**4.** Adjust the launch angle to 10º and insert the plastic ball into the second stage.

**5.** Fire launches to locate the point where the ball touches the wall. Attach at this point a sheet of carbon paper.

**6.** Fire the launch and measure the height (from the floor) reached by the projectile.

**7.** Launch at the angles suggested in table 1 and note the respective heights reached.

Table 1

| *Angle $\theta$ (degree)* | *10* | *15* | *20* | *25* | *30* | *35* | *40* | *45* | *50* | *55* | *60* | *65* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *Height y (m)* | | | | | | | | | | | | |

>>> ***Analysis of Results and Conclusions*** >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

**8.** Note in table 2 the launch velocity value obtained in experiment 1.

**9.** Use the initial velocity obtained in experiment 1.

**10.** By using the initial velocity of step 1 and the distance from the wall to the launcher, calculate the angle that provides the maximum height.

**11.** Draw the graph of the vertical position versus the launch angle of table 1.

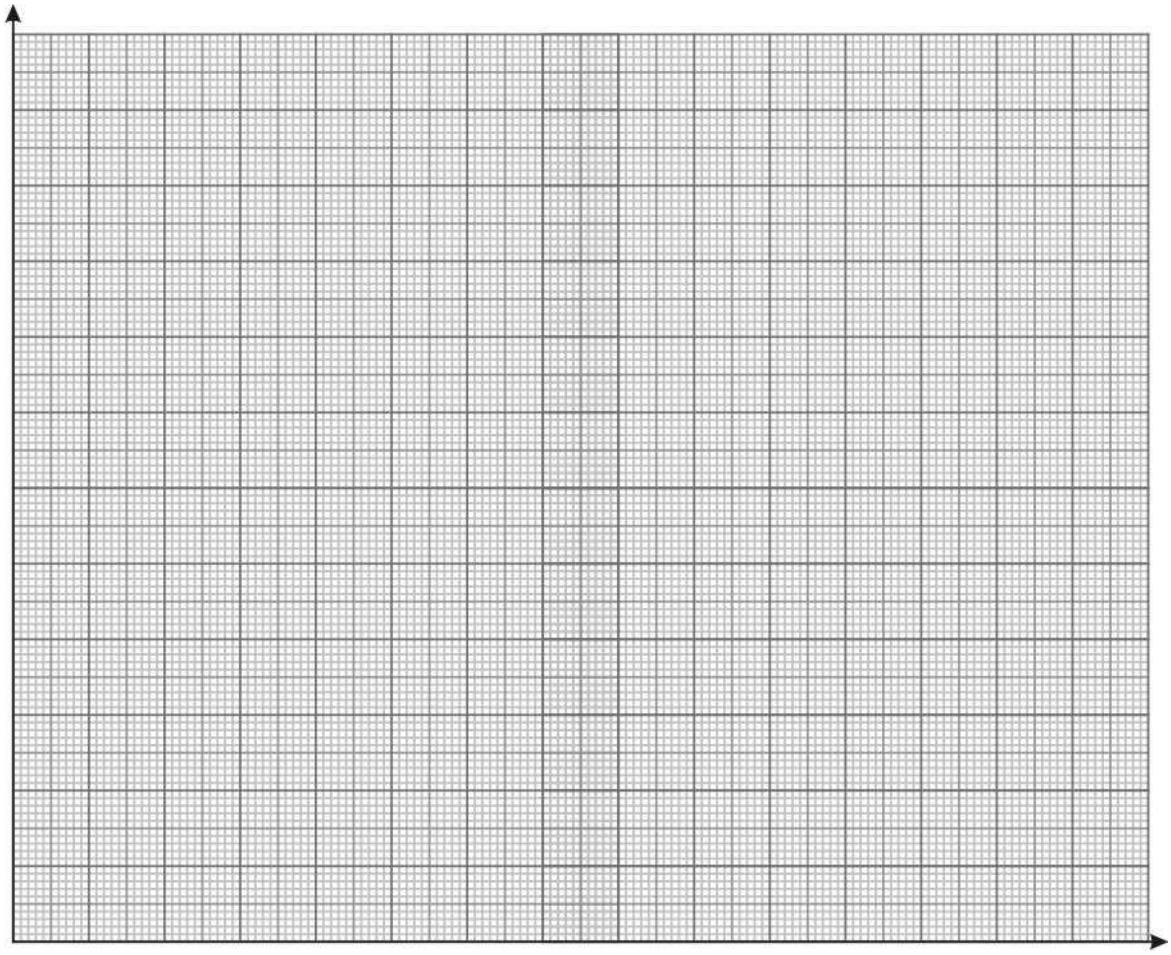


Table 2

| | *Valor* |
|---|---|
| *Angle for maximum height-measured* | |
| *Maximum height* | |
| *Horizontal wall distance* | |
| *Launch height* | |
| *Initial velocity calculated* | |
| *Angle for maximum height-calculated* | |
| *Percentage difference among angles* | |

**12.** Does the value found analytically agree with the values obtained experimentally? Justify.

**13.** For the angle that gives the maximum height when the ball hits the wall, has it reached the peak of the trajectory?

**14.** How far from the wall will the height be maximized for a launch at a 45° angle? What would be the maximum height in this case?

**15.** Launch to x equal to the value found for the 45° angle and measure the y value (x; 45°). Compare with calculated value.

# EXPERIMENT 10 – BALLISTIC PENDULUM - APPROXIMATE METHOD

**Objectives:**
**- Using the conservation of linear momentum and conservation of mechanical energy in a ballistic pendulum to find the launch velocity of a projectile.**
**- Comparing the launch velocity found with the velocity obtained in a horizontal launch by using different processes.**

>>> ***Material Used*** >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

*PROJECTILE LAUNCHER*

| *Item* | *Code* | *Quant.* | *Unit* | *Description* |
|---|---|---|---|---|
| *01* | *62002176* | *01* | *Un.* | *CLAMP "C"* |
| *02* | *62005751* | *01* | *Un.* | *CANNON LAUNCHER HOLDER* |
| *03* | *62002015* | *01* | *Un.* | *CANNON (PROJECTILE LAUNCHER)* |
| *04* | *62001074* | *01* | *Un.* | *STEEL BALL Ø25MM* |
| *06* | *62005177* | *01* | *Un.* | *TUBE FOR CANNON COMPRESSION* |
| *07* | *48005003* | *02* | *Un.* | *BUTTERFLY NUT (CANNON'S FASTENER)* |
| *08* | *50001004* | *02* | *Un.* | *FLAT WASHER (CANNON'S FASTENER)* |
| *09* | *62005317* | *01* | *Un.* | *CANNON'S FASTENER* |
| *13* | *03003011* | *01* | *Un.* | *TAPE MEASURE 05M* |
| *14* | *62005274* | *01* | *Un.* | *PLUMB BOB WITH MAGNETIC FIXING* |

*Ballistic Pendulum Accessories(sold separately)*

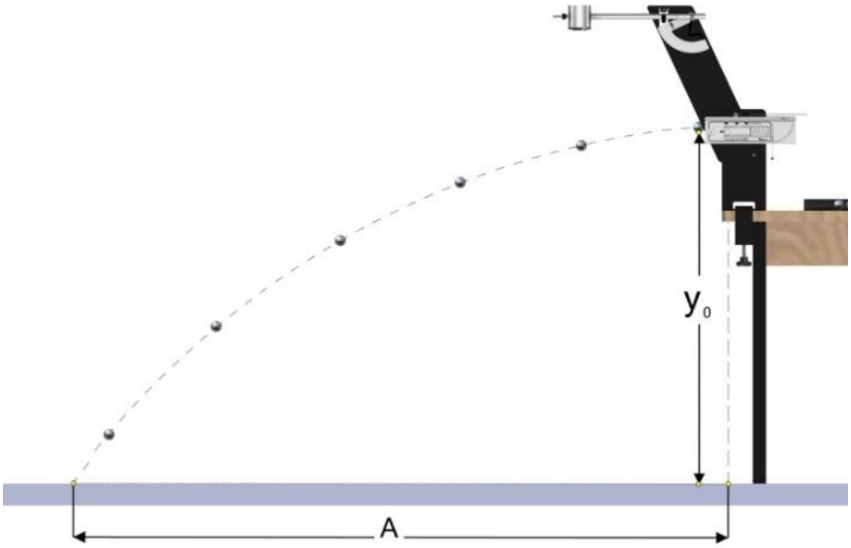
| *Item* | *Code* | *Quant.* | *Unit* | *Description* |
|---|---|---|---|---|
| *01* | *62005611* | *02* | *Un.* | *50G CYLINDER MASS* |
| *03* | *53003001* | *01* | *Un.* | *PENDULUM'S TOWER THUMB SCREW'S NUT* |
| *04* | *53001009* | *01* | *Un.* | *PENDULUM'S THUMB SCREW* |
| *04* | *53004002* | *01* | *Un.* | *METAL HANDLE M3X10* |
| *05* | *62002055* | *01* | *Un.* | *BALLISTIC PENDULUM* |
| *06* | *62002182* | *01* | *Un.* | *PENDULUM'S TOWER* |

## Part I - Obtaining the horizontal launch velocity by using the reach measure.

>>> ***Experimental Procedures***>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

Do not look directly at the cannon exit, as it may be charged!

**1.** Assemble the ballistic pendulum as shown.




**2.** Measure the launch height (yo) in relation to the floor. The measurement shall be made from the lower part of the ball indicated at the exit of the cannon, to the floor, according to the figure.

**3.** With a plumb bob, mark on a sheet of A4 paper pasted with adhesive tape on the floor, the post position (origin of horizontal displacement). The plumb bob must match the vertical passing through the center of the ball.

**4.** Place the steel ball in the projectile launcher and compress the spring to the third stage. Pull the trigger and observe where the ball touches the floor.

**5.** At the dropping point of the ball, affix a paper and on it a sheet of carbon paper for marking the projectile reach distance A.

**6.** Repeat the launch 5 times and measure the reach distance A.

Table 1

| N | Reach (m) |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| Average value | |

**7.** Use the horizontal launch equations and calculate the launch velocity:

# Part II - Obtaining launch velocity by using the conservation of linear momentum in a ballistic pendulum

**>>> *Experimental Procedures*>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>**

Do not look directly at the cannon exit, as it may be charged!

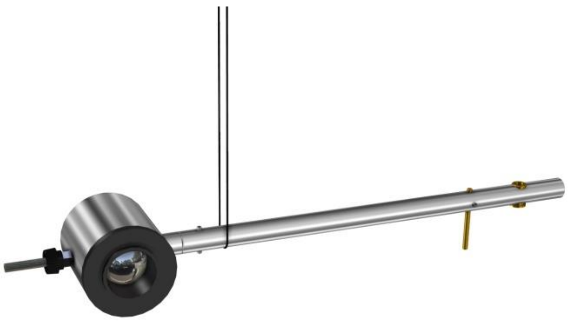
1. Use the 1st part launcher assembly at an end of the table.

2. Measure the mass m of the metal ball and the mass M of the set (ballistic pendulum + ball).

   m=0,064 kg e M = 0,294 kg

3. Insert the ball into the pendulum receiver and use a line to find the center of mass position of the set. Slide the line holding the pendulum until it remains in horizontal balance.

4. Measure the distance R from the center of mass position to the axis of rotation.




5. Attach the pendulum to the launcher support and make the necessary adjustments.

6. For the correct positioning of the pendulum the following steps are suggested:
   - Keep the pendulum free in vertical.
   - Carefully approach the cannon until it softly engages the mouth of the pendulum trimmer.
   - Tighten the cylinder locking screws.

   With these procedures the ball, when launched, will always be correctly picked up by the pendulum receiver.

7. Place the steel ball in the projectile launcher and compress the spring to the second stage.

**8.** Adjust the pendulum position so the axis of trimmer fits correctly into the launcher's mouth.

**9.** Adjusting the zero of the angle marker.

**10.** Release the trigger for launch and note the angle value reached by the pendulum.

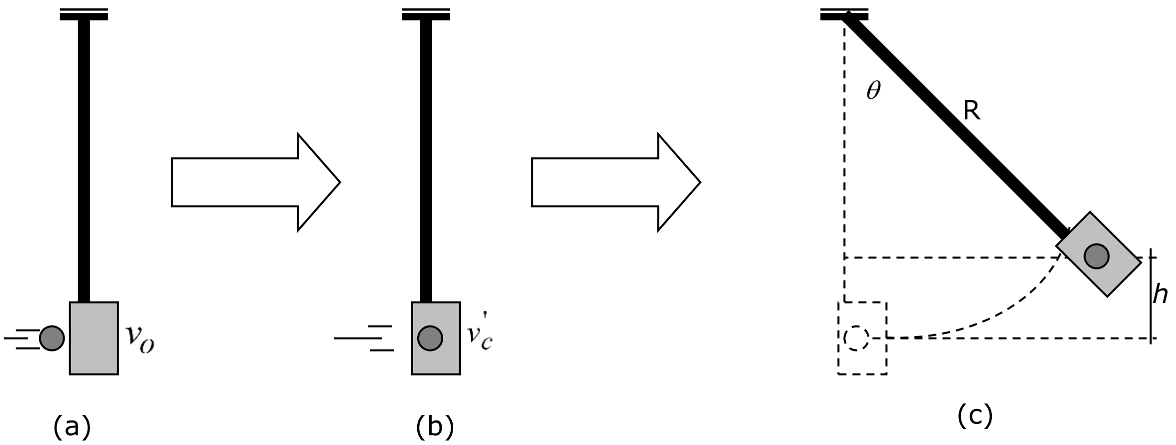
**11.** Write down the measurements of angles of the pendulum.

**12.** Choose the most repeatable measure and adjust the indicator to about four degrees less than that angle.

**13.** Repeat the launch three times and calculate the average value of the angle.

Table 1

| $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_{average}$ |
|---|---|---|---|
| | | | |




**>>>Theoretical Foundations>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>**

The height *h* reached by the center of mass of the pendulum is calculated by:

$$\mathbf{h = R(1 - cos\theta)}$$

Considering that the linear momentum of the system is conserved both in an elastic collision and in an inelastic collision, we have:

$$p_{before\ collision} = p_{after\ collision} \rightarrow mv_0 = M \cdot v \quad (1)$$

After the collision the ballistic pendulum moves and its center of mass reaches a height h:
As mechanical energy is conserved in this movement, we have:

$$K_{after\ collision} = U_{max\ hieght} \rightarrow \frac{Mv^2}{2} = MgR(1 - cos\theta) \rightarrow v = \sqrt{2 \cdot g \cdot R(1 - cos\theta)} \quad (2)$$

By combining (1) and (2), we obtain the value of the launch velocity before the pendulum collision ($v_o$):

$$v_o = \frac{M}{m}\sqrt{2 \cdot g \cdot R(1 - cos\theta)}$$

### >>> ***Analysis of Results and Conclusions*** >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

**14.** Use the deduced equation to calculate the ball launch velocity value.

**15.** Compare the result found with the average velocity of the ball obtained in the horizontal launch in the first part.

**16.** Calculate the percentage difference between the two results found: $d\% = \frac{|A-B|}{\frac{A+B}{2}} \times 100\%$

**17.** Does the percentage difference obtained confirm the validity of the conservation principle of linear momentum? Justify.

**18.** What can be concluded about the energy conservation in the collision?

# EXPERIMENT 11 – BALLISTIC PENDULUM-EXACT METHOD

**Objectives:**
**- Using the conservation of angular momentum and conservation of mechanical energy in a ballistic pendulum to find the launch velocity of a projectile.**
**- Using the conservation of linear momentum and conservation of mechanical energy in a ballistic pendulum to find the launch velocity of a projectile.**

>>> ***Material Used*** >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

## *PROJECTILE LAUNCHER*

| *Item* | *Code* | *Quant.* | *Unit* | *Description* |
|---|---|---|---|---|
| *01* | *62002176* | *01* | *Un.* | *CLAMP "C"* |
| *02* | *62005751* | *01* | *Un.* | *CANNON LAUNCHER HOLDER* |
| *03* | *62002015* | *01* | *Un.* | *CANNON (PROJECTILE LAUNCHER)* |
| *04* | *62001074* | *01* | *Un.* | *STEEL BALL Ø25MM* |
| *06* | *62005177* | *01* | *Un.* | *TUBE FOR CANNON COMPRESSION* |
| *07* | *48005003* | *02* | *Un.* | *BUTTERFLY NUT (CANNON'S FASTENER)* |
| *08* | *50001004* | *02* | *Un.* | *FLAT WASHER (CANNON'S FASTENER)* |
| *09* | *62005317* | *01* | *Un.* | *CANNON'S FASTENER* |
| *13* | *03003011* | *01* | *Un.* | *TAPE MEASURE 05M* |
| *14* | *62005274* | *01* | *Un.* | *PLUMB BOB WITH MAGNETIC FIXING* |

## *Ballistic Pendulum Accessories(sold separately)*

| *Item* | *Code* | *Quant.* | *Unit* | *Description* |
|---|---|---|---|---|
| *01* | *62005611* | *02* | *Un.* | *50G CYLINDER MASS* |
| *03* | *53003001* | *01* | *Un.* | *PENDULUM'S TOWER THUMB SCREW'S NUT* |
| *04* | *53001009* | *01* | *Un.* | *PENDULUM'S THUMB SCREW* |
| *04* | *53004002* | *01* | *Un.* | *METAL HANDLE M3X10* |
| *05* | *62002055* | *01* | *Un.* | *BALLISTIC PENDULUM* |
| *06* | *62002182* | *01* | *Un.* | *PENDULUM'S TOWER* |

>>> ***Experimental Procedures***>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

Do not look directly at the cannon exit, as it may be charged!

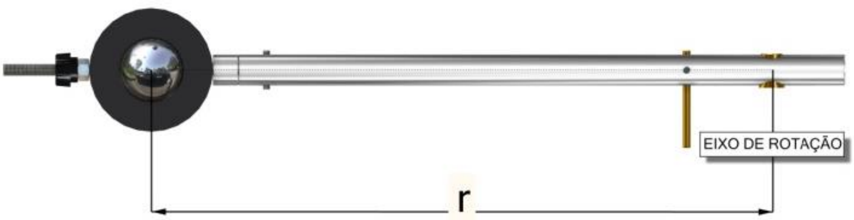
**1.** Assemble the launcher as in the previous experiment.

**2.** Measure the mass m of the metal ball and the mass M of the set (ballistic pendulum + ball).

*m = 0,064 kg   M = 0,294 kg*

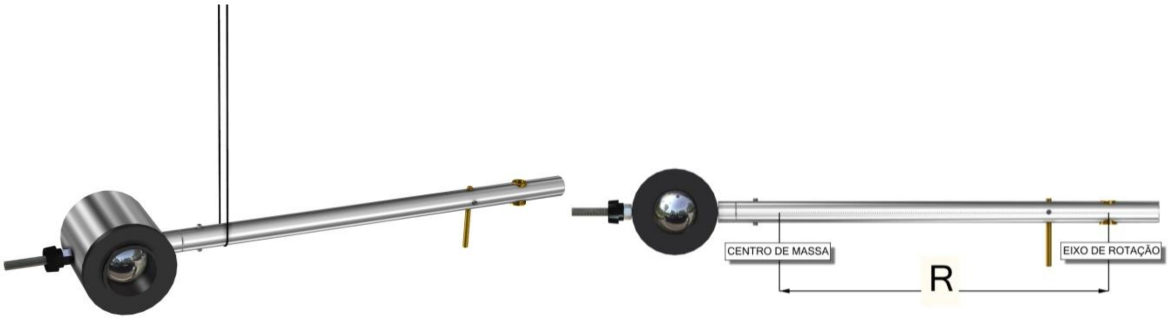
**3.** Measure the distance r from the center of the ball to the axis of rotation of the pendulum.

*r =0,255 m*



**4.** Insert the ball into the pendulum receiver and use a line to find the center of mass position of the set. Slide the line holding the pendulum until it remains in horizontal balance.

**5.** Measure the distance R from the position of the center of mass to the axis of rotation.




**6.** Attach the pendulum to the launcher support and make the necessary adjustments.

**7.** For the correct positioning of the pendulum the following steps are suggested:

- Keep the pendulum free in vertical.
- Carefully approach the cannon until it softly engages the mouth of the pendulum trimmer.
- Tighten the cylinder locking screws.

With these procedures the ball, when launched, will always be correctly picked up by the pendulum receiver.

**8.** Place the steel ball in the projectile launcher and compress the spring to the second stage.

**9.** Adjust the pendulum position to align correctly into the launcher's mouth.

**10.** Adjusting the zero of the angle marker.

**11.** Trigger the cannon and note the angle value reached by the pendulum.

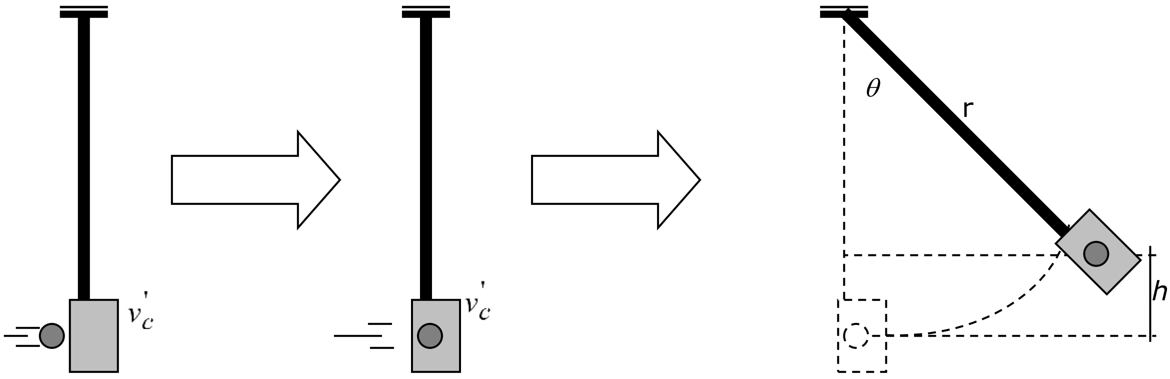
**12.** Fire some measurements of the reach angle of the pendulum.

**13.** Choose the most repeated measure and adjust the indicator to about four degrees less than that angle.

**14.** Repeat the launch three times and calculate the average value of the angle.

Table 1

| $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_{average}$ |
|---|---|---|---|
| | | | |




(a) (b) (c)

**15.** Assemble the pendulum without the launcher so it can swing freely.

**16.** Use a timer and measure the time of 20 complete oscillations of small amplitude. Repeat the procedure for at least three times and note the time value (t = 20T) for the 20 oscillations in the table.

**>>>Theoretical Foundations>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>**

The height *h* reached by the center of mass of the pendulum is as shown in (c):

$$\boldsymbol{h = R(1 - cos\theta)}$$

The potential energy stored in the system at the (c)

$$\boldsymbol{U = M \cdot g \cdot h = MgR(1 - cos\theta)}$$

The kinetic energy immediately after the collision equals the potential energy at the point where the pendulum reaches the largest angle, and therefore:

$$\boldsymbol{U = K_d = M \cdot g \cdot h = MgR(1 - cos\theta)}(1)$$

The kinetic energy $K_d$ and the angular momentum $L_d$ of the system immediately after the collision of the ball with the pendulum are given by the equations:

$\boldsymbol{K_d = \frac{I \cdot \omega^2}{2}}$ and $\boldsymbol{L_d = I_{conj} \cdot \omega}$

By combining the two equations we can obtain the relation between $L_d$ and $K_d$:

$$\boldsymbol{L_d = \sqrt{2 \cdot I_{conj} \cdot K_d}} \qquad (2)$$

The angular momentum of the system immediately before the collision ($L_a$) is restricted to the angular momentum of the ball, since at that moment the pendulum is at rest:

$$\boldsymbol{L_a = I_{ball} \cdot \omega = m \cdot r^2 \omega = m \cdot r \cdot v_0} \quad (3)$$

$$\text{Where:}\begin{cases} \boldsymbol{m - mass\ of\ the\ ball} \\ \boldsymbol{r - distance\ from\ the\ ball\ to\ the\ axis\ of\ rotation} \\ \boldsymbol{v_0 - velocity\ of\ the\ ball\ before\ the\ collision} \end{cases}$$

Considering that the angular momentum L of the system is conserved and by combining (2) and (3):

$$\boldsymbol{L_{before\ collision} = L_{after\ collision}}$$

$$\boldsymbol{m \cdot r \cdot v_0 = \sqrt{2 \cdot I_{set} \cdot K_d}} \qquad (4)$$

By combining (1) and (4), we obtain: $\boldsymbol{v_0 = \frac{1}{m \cdot r}\sqrt{2 \cdot I_{set} \cdot M \cdot g \cdot R(1 - cos\theta)}}$ (5)

In the angular displacement of the pendulum the torque: $\boldsymbol{\tau = I_{set} \cdot \alpha = -R \cdot M \cdot g \cdot sen\theta}$

For small angles this equation can be written: $\boldsymbol{\alpha = \frac{d^2\theta}{dt^2} = -\frac{MgR}{I_{set}} \cdot \theta}$

Which is similar to a simple harmonic motion equation $\boldsymbol{\alpha = -\frac{k}{m}x = -\omega^2 x}$

What allows us to write: $\boldsymbol{\omega^2 = \frac{MgR}{I} \rightarrow I_{conj} = \frac{MgR}{\omega^2} \rightarrow I_{conj} = \frac{MgRT^2}{4\pi^2}}$(6)

The period T can be obtained by making the pendulum perform small oscillations.

### >>> ***Analysis of Results and Conclusions*** >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

**17.** Calculate the average value of the period T of the pendulum.

Table 2

| *Event* | *1* | *2* | *3* | *Average value* |
|---|---|---|---|---|
| *Time of 20 oscillations (s)* | | | | |
| *Period T (s)* | | | | |

**18.** Use the equation (6) and calculate the moment of inertia of the set $I_{set}$.

**19.** Use the equation (5) and calculate the ball velocity immediately before launch.

**20.** Calculate the percentage difference between the two results found: $\boldsymbol{d\% = \frac{|A-B|}{\frac{A+B}{2}} \times 100\%}$

**21.** Does the percentage difference obtained confirm the validity of the conservation principle of angular momentum? Justify.

**22.** What can be concluded about the energy conservation in the collision?